

Scilab Textbook Companion for
Linear Algebra And Its Applications
by G. Strang¹

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Book Description

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Scilab numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

AP Appendix to Example(Scilab Code that is an Appednix to a particular Example of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means a scilab code whose theory is explained in Section 2.3 of the book.

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Chapter 1

Matrix Notation and Matrix Multiplication

Scilab code Exa 1.3.1 Breakdown of elimination

```
1 clear;
2 close;
3 clc;
4 a =[1 1 1;2 2 5;4 6 8]
5 disp('x=[u;v;w]')
6 disp('R2=R2-R1,R3=R3-4*R1')
7 a(2,:)=a(2,)-2*a(1,);
8 a(3,:)=a(3,)-4*a(1,);
9 disp(a);
10 disp('R2<->R3')
11 b=a(2,);
12 a(2,:)=a(3,);
13 a(3,:)=b;
14 disp(a);
15 disp('The system is now triangular and the equations
      can be solved by Back substitution');
16 //end
```

Scilab code Exa 1.3.2 Breakdown of elimination

```
1 clear ;
2 close ;
3 clc ;
4 a =[1 1 1;2 2 5;4 4 8];
5 disp(a, 'a=');
6 disp('x=[u;v;w] ');
7 disp('R2=R2-2*R1, R3=R3-4*R1 ');
8 a(2,:)=a(2,:)-2*a(1,:);
9 a(3,:)=a(3,:)-4*a(1,:);
10 disp(a);
11 disp('No exchange of equations can avoid zero in the
      second pivot position ,therefore the equations
      are unsolvable ');
12 //end
```

Scilab code Exa 1.4.1 Multiplication of Two Matrices

```
1 clear ;
2 close ;
3 clc ;
4 A=[2 3;4 0];
5 disp(A, 'A=');
6 B=[1 2 0;5 -1 0];
7 disp(B, 'B');
8 disp(A*B, 'AB=')
9 //end
```

Scilab code Exa 1.4.2 Multiplication with Row exchange matrix

```
1 clear;
2 close;
3 clc;
4 A=[2 3;7 8];
5 disp(A, 'A=');
6 P=[0 1;1 0];
7 disp(P, 'P(Row exchange matrix)=')
8 disp(P*A, 'PA=')
9 //end
```

Scilab code Exa 1.4.3 Multiplication with Identity Matrix

```
1 //page 24
2 clear;
3 close;
4 clc;
5 A=[1 2;3 4];
6 disp(A, 'A=');
7 I=eye(2,2);
8 disp(I, 'I=');
9 disp(I*A, 'IA=')
10 //end
```

Scilab code Exa 1.4.4 Matrix multiplication not commutative

```
1 //page 25
2 clear;
3 close;
4 clc;
5 E=eye(3,3);
6 E(2,:)=E(2,:)-2*E(1,:);
```

```

7 disp(E, 'E=');
8 F=eye(3,3);
9 F(3,:)=F(3,:)+F(1,:);
10 disp(F, 'F=');
11 disp(E*F, 'EF=')
12 disp(F*E, 'FE=')
13 disp('Here ,EF=FE,so this shows that these two
      matrices commute')
14 //end

```

Scilab code Exa 1.4.5 Order of Elimination

```

1 //page 25
2 clear;
3 close;
4 clc;
5 E=eye(3,3);
6 E(2,:)=E(2,:)-2*E(1,:);
7 disp(E, 'E')
8 F=eye(3,3);
9 F(3,:)=F(3,:)+F(1,:);
10 disp(F, 'F=');
11 G=eye(3,3);
12 G(3,:)=G(3,:)+G(2,:);
13 disp(G, 'G')
14 disp(G*E, 'GE=')
15 disp(E*G, 'EG=')
16 disp('Here EG is not equal to GE,Therefore these two
      matrices do not commute and shows that most
      matrices do not commute.')
```

```

17 disp(G*F*E, 'GFE=')
18 disp(E*F*G, 'EFG=')
19 disp('The product GFE is the true order of elimination
      .It is the matrix that takes the original A to
      the upper triangular U.')
```

20 //end

Scilab code Exa 1.5.1 Triangular factorization

```
1 //page 34
2 clear;
3 close;
4 clc;
5 A=[1 2;3 8];
6 disp(A, 'A=');
7 [L,U]=lu(A);
8 disp(L, 'L=');
9 disp(U, 'U=');
10 disp('LU=')
11 disp(L*U)
12 disp('This shows that LU=A')
13 //end
```

Scilab code Exa 1.5.2 To check LU equals to A

```
1 //page 34
2 clear;
3 close;
4 clc;
5 A=[0 2;3 4];
6 disp(A, 'A=')
7 disp('Here this cannot be factored into A=LU,(Needs
      a row exchange)');
8 //end
```

Scilab code Exa 1.5.3 To check LU equals to A

```
1 //page 34
2 clear;
3 close;
4 clc;
5 disp('Given Matrix:')
6 A=[1 1 1;1 2 2;1 2 3];
7 disp(A, 'A=');
8 [L,U]=lu(A);
9 disp(L, 'L=');
10 disp(U, 'U=');
11 disp(L*U, 'LU=');
12 disp('Here LU=A,from A to U there are subtraction of
      rows.Frow U to A there are additions of rows');
13 //end
```

Scilab code Exa 1.5.4 If U equals to I then L equals to A

```
1 //page 34
2 clear;
3 close;
4 clc;
5 a=rand(1);
6 b=rand(1);
7 c=rand(1);
8 L=[1 0 0;a 1 0;b c 1];
9 disp(L, 'L=');
10 U=eye(3,3);
11 disp(U, 'U=');
12 E=[1 0 0;-a 1 0;0 0 1];
13 disp(E, 'E=');
14 F=[1 0 0;0 1 0;-b 0 1];
15 disp(F, 'F=');
16 G=[1 0 0;0 1 0;0 -c 1];
```

```

17 disp(G, 'G=');
18 disp('A=inv(E)*inv(F)*inv(G)*U')
19 A=inv(E)*inv(F)*inv(G)*U;
20 disp(A, 'A=');
21 disp('When U is identity matrix then L is same as A'
      ');
22 //end

```

Scilab code Exa 1.5.5 Spilting A to L and U

```

1 //page 39
2 clear;
3 close;
4 clc;
5 A=[1 -1 0 0 ;-1 2 -1 0;0 -1 2 -1;0 0 -1 2];
6 disp(A, 'A=');
7 [L,U]=lu(A);
8 disp(U, 'U=');
9 disp(L, 'L=');
10 disp('This shows how a matrix A with 3 diagnols has
      factors L and U with two diagnols.')
11 //end

```

Scilab code Exa 1.5.6 Solving for X using L and U

```

1 //page 36
2 clear;
3 close;
4 clc;
5 a=[1 -1 0 0;-1 2 -1 0;0 -1 2 -1;0 0 -1 2];
6 disp(a, 'a=')
7 b=[1;1;1;1]
8 disp(b, 'b=')

```

```

9  disp('Given Equation ,ax=b')
10 [L,U]=lu(a);
11 disp(U, 'U=');
12 disp(L, 'L=');
13 disp('Augmented Matrix of L and b=');
14 A=[L b];
15 disp(A)
16 c=zeros(4,1);
17 n=4;
18 //Algorithm Finding the value of c
19 c(1)=A(1,n+1)/A(1,1);
20 for i=2:n;
21     sumk=0;
22     for k=1:n-1
23         sumk=sumk+A(i,k)*c(k);
24     end
25     c(i)=(A(i,n+1)-sumk)/A(i,i)
26 end
27 disp(c, 'c=');
28 x=zeros(4,1);
29 disp('Augmented matrix of U and c=')
30 B=[U c];
31 disp(B)
32 //Algorithm for finding value of x
33 x(n)=B(n,n+1)/B(n,n);
34 for i=n-1:-1:1;
35     sumk=0;
36     for k=i+1:n
37         sumk=sumk+B(i,k)*x(k);
38     end
39     x(i)=(B(i,n+1)-sumk)/B(i,i);
40 end
41 disp(x, 'x=')
42 //end

```

Scilab code Exa 1.5.7 Elimination in a nutshell

```
1 //page 39
2 clear;
3 close;
4 clc;
5 A=[1 1 1;1 1 3;2 5 8];
6 disp(A, 'A=');
7 [L,U,P]=lu(A);
8 disp(L, 'L=');
9 disp(U, 'U=');
10 disp(P, 'P=');
11 disp(P*A, 'PA=')
12 disp(L*U, 'LU=')
13 disp('This shows that PA is the same as LU')
14 //end
```

Scilab code Exa 1.6.1 Gauss Jordan method

```
1 //page 47
2 clear;
3 close;
4 clc;
5 disp('Given matrix:')
6 A=[2 1 1;4 -6 0;-2 7 2];
7 disp(A);
8 [n,m]=size(A);
9 disp('Augmented matrix :')
10 a=[A eye(n,m)];
11 disp(a)
12 disp('R2=R2-2*R1, R3=R3-(-2)*R1');
13 a(2,:)=a(2,:)-2*a(1,:);
14 a(3,:)=a(3,:)-(-1)*a(1,:);
15 disp(a)
16 disp('R3=R3-(-1)*R2');
```



```

17 a(3,:) = a(3,:) - (-1)*a(2,:);
18 disp(a, 'a=')
19 disp(a, '[U inv(L)] :')
20 disp('R2=R2-(-2)*R3, R1=R1-R3')
21 a(2,:) = a(2,:) - (-2)*a(3,:);
22 a(1,:) = a(1,:) - a(3,:);
23 disp(a)
24 disp('R1=R1-(-1/8)*R2')
25 a(1,:) = a(1,:) - (-1/8)*a(2,:);
26 disp(a)
27 a(1,:) = a(1, :)/a(1, 1);
28 a(2,:) = a(2, :)/a(2, 2);
29 disp(' [I inv(A)] :')
30 a(3,:) = a(3, :)/a(3, 3);
31 disp(a);
32 disp('inv(A) :')
33 a(:, 4:6);
34 disp(a(:, 4:6))

```

Scilab code Exa 1.6.2 Symmetric products

```

1 //Caption :Symmetric Products
2 //Example:1.6.2 -To Find the product of transpose(R)
  and R.
3 //page 51
4 clear;
5 close;
6 clc;
7 R=[1 2];
8 disp(R, 'R=');
9 Rt=R';
10 disp(Rt, 'Transpose of the given matrix is :')
11 disp(R*Rt, 'The product of R & transpose(R) is :')
12 disp(Rt*R, 'The product of transpose(R)& R is :')
13 disp('Rt*R and R*Rt are not likely to be equal even

```

```
    if m==n. ')
14 //end
```

Chapter 2

Vector Spaces

Scilab code Exa 2.1.1 Vector Spaces and subspaces

```
1 //page 70
2 clear;
3 close;
4 clc;
5 disp('Consider all vectors in R^2 whose components
      are positive or zero')
6 disp('The subset is first Quadrant of x-y plane ,the
      co-ordinates satisfy x>=0 and y>=0.It is not a
      subspace.')
```

```
7 v=[1,1];
8 disp(v,'If the Vector=');
9 disp('Taking a scalar ,c=-1')
10 c=-1; //scalar
11 disp(c*v,'c*v=')
12 disp('It lies in third Quadrant instead of first ,
      Hence violating the rule(ii).')
```

```
13 //end
```

Scilab code Exa 2.1.2 Vector Spaces and subspaces

```

1 //page 71
2 clear;
3 close;
4 clc;
5 disp('Take vector space of 3X3 matrices')
6 disp('One possible subspace is the set of lower
    triangular matrices ,Another is set of symmetric
    matrices')
7 disp('A+B,cA are both lower triangular if A and B
    are lower triangular ,and are symmetric if A and
    B are symmetric and Zero matrix is in both
    subspaces')

```

Scilab code Exa 2.3.1 Linear Independence

```

1 //page 92
2 clear;
3 close;
4 clc;
5 disp('For linear independence , $C_1V_1+C_2V_2+\dots+C_kV_k=0$ 
    ')
6 disp('If we choose  $V_1$ =zero vector ,then the set is
    linearly dependent.We may choose  $C_1=3$  and all
    other  $C_i=0$ ;this is a non-trivial solution that
    produces zero.')
7 //end

```

Scilab code Exa 2.3.2 Linear Independence

```

1 //page 92
2 clear;
3 close;
4 clc;

```

```

5 A=[1 3 3 2;2 6 9 5;-1 -3 3 0];
6 disp('Given matrix:')
7 disp(A)
8 B=A;
9 disp('C2->C2-3*C1')
10 A(:,2)=A(:,2)-3*A(:,1);
11 disp(A)
12 disp('Here ,C2=3*C1, Therefore the columns are
        linearly dependent. ')
13 disp('R3->R3-2*R2+5*R1')
14 B(3,:)=B(3,:)-2*B(2,:)+5*B(1,:);
15 disp(B)
16 disp('Here R3=R3-2*R2+5*R1, therefore the rows are
        linearly dependent. ')
17 //end

```

Scilab code Exa 2.3.3 Linear Independence

```

1 clear;
2 close;
3 clc;
4 A=[3 4 2;0 1 5;0 0 2];
5 disp(A, 'A=');
6 disp('The columns of the triangular matrix are
        linearly independent, it has no zeros on the
        diagonal');
7 //end

```

Scilab code Exa 2.3.4 Linear Independence

```

1 //page 93
2 clear;
3 close;

```

```

4 clc;
5 disp('The columns of the nxn identity matrix are
      independent. ')
6 n=input('Enter n:');
7 I=eye(n,n);
8 disp(I, 'I=');
9 //end

```

Scilab code Exa 2.3.5 Linear Independence

```

1 //page 93
2 clear;
3 close;
4 clc;
5 disp('Three columns in R2 cannot be independent. ')
6 A=[1 2 1;1 2 3];
7 disp(A, 'Given matrix: ')
8 [L,U]=lu(A);
9 disp(U, 'U=');
10 disp('If c3 is 1 ,then back-substitution Uc=0 gives
      c2=-1,c1=1,With these three weights ,the first
      column minus the second plus the third equals
      zero ,therefore linearly dependent. ')

```

Scilab code Exa 2.3.6 Linear Independence

```

1 //page 93
2 clear;
3 close;
4 clc;
5 disp('The vectors  $w_1=(1,0,0)$ ,  $w_2=(0,1,0)$ ,  $w_3=(-2,0,0)$ 
      span a plane (x-y plane) in R3. The first two

```

```
        vectors also span this plane , whereas w1 and w3
        span only a line. ');
6 //end
```

Scilab code Exa 2.3.7 Linear Independence

```
1 //page 93
2 clear;
3 close;
4 clc;
5 disp('The column space of A is excatly the space
        that is spanned by its columns.The row space is
        spanned by the rows.The definition is made to
        order.Multiplying A by any x gives a combination
        of columns; it is a vector Ax in the column space
        . The coordinate vectors e_1 ,....e_n coming from
        the identity matrix span Rn. Every vector b=(b_1
        ..... ,b_n) is a combination of those columns.In
        this example the weights are the components b_i
        themselves:b=b_1e_1+.....+b_ne_n.But the columns
        of other matrices also span R..')
6 //end
```

Scilab code Exa 2.3.8 Basis for a vector space

```
1 //page 93
2 clear;
3 close;
4 clc;
5 disp('Here, the vector v1 by itself is linearly
        independent , but it fails to span R2.The three
        vectors v1,v2,v3 certainly span R2, but are not
        independent. Any two of these vectors say v1 and
```

```
    v2 have both properties –they span and they are
    independent. So they form a basis. (A vector space
    does not have a unique basis)')
6 //end
```

Scilab code Exa 2.3.9 Basis for a vector space

```
1 //page 96
2 clear;
3 close;
4 clc;
5 disp('These four columns span the column space U, but
    they are not independent.')
```

```
6 U=[1 3 3 2;0 0 3 1;0 0 0 0];
7 disp(U, 'U=');
8 disp('The columns that contains pivots (here 1st & 3
    rd) are a basis for the column space. These
    columns are independent, and it is easy to see
    that they span the space. In fact, the column space
    of U is just the x-y plane within R3. C(U) is
    not the same as the column space C(A) before
    elimination –but the number of independent columns
    did not change.')
```

Scilab code Exa 2.4.1 The four fundamental subspaces

```
1 //page 107
2 clear;
3 close;
4 clc;
5 A=[1 2;3 6];
6 disp(A, 'A=');
7 [m,n]=size(A);
```



```

8  disp(m, 'm=');
9  disp(n, 'n=');
10 [v,pivot]=rref(A);
11 r=length(pivot);
12 disp(r, 'rank=')
13 cs=A(:,pivot);
14 disp(cs, 'Column space=');
15 ns=kernel(A);
16 disp(ns, 'Null space=');
17 rs=v(1:r,:)' ;
18 disp(rs, 'Row space=')
19 lns=kernel(A');
20 disp(lns, 'Left null sapce=');

```

Scilab code Exa 2.4.2 Inverse of a mxn matrix

```

1  //page 108
2  clear;
3  close;
4  clc;
5  A=[4 0 0;0 5 0];
6  disp(A, 'A=');
7  [m,n]=size(A);
8  disp(m, 'm=');
9  disp(n, 'n=')
10 r=rank(A);
11 disp(r, 'rank=');
12 disp('since m=r=2 ,there exists a right inverse .');
13 C=A'*inv(A*A');
14 disp(C, 'Best right inverse=')
15 //end

```

Scilab code Exa 2.5.1 Networks and discrete applied mathematics

```

1 //page 121
2 clear;
3 close;
4 clc;
5 disp('Applying current law A''y=f at nodes 1,2,3:')
6 A=[-1 1 0;0 -1 1; -1 0 1;0 0 -1;-1 0 0];
7 disp(A', 'A''='');
8 C=diag(rand(5,1)); //Taking some values for the
    resistances.
9 b=zeros(5,1);
10 b(3,1)=rand(1); //Taking some value of the battery.
11 f=zeros(3,1);
12 f(2,1)=rand(1); //Taking some value of the current
    source.
13 B=[b;f];
14 disp('The other equation is inv(C)y+Ax=b.The block
    form of the two equations is:')
15 C=[inv(C) A;A' zeros(3,3)];
16 disp(C);
17 X=['y1 ','y2 ','y3 ','y4 ','y5 ','x1 ','x2 ','x3 '];
18 disp(X, 'X=')
19 X=C\B;
20 disp(X, 'X=');
21 //end

```

Chapter 3

Orthogonality

Scilab code Exa 3.1.1 Orthogonal vectors

```
1 //page 143
2 clear;
3 close;
4 clc;
5 x1=[2;2;-1];
6 disp(x1, 'x1=');
7 x2=[-1;2;2];
8 disp(x2, 'x2=');
9 disp(x1'*x2, 'x1''*x2=');
10 disp('Therefore, X1 is orthogonal to x2 .Both have
      length of sqrt(14).')
```

Scilab code Exa 3.1.2 Orthogonal vectors

```
1 //page 144
2 clear;
3 close;
4 clc;
```

```
5 disp('Suppose V is a plane spanned by v1=(1,0,0,0)
    and v2=(1,1,0,0). If W is the line spanned by w
    =(0,0,4,5), then w is orthogonal to both v's. The
    line W will be orthogonal to the whole plane V.')
```

Scilab code Exa 3.1.3 Orthogonal vectors

```
1 //page 145
2 clear;
3 close;
4 clc;
5 A=[1 3;2 6;3 9];
6 disp(A, 'A=');
7 ns=kernel(A);
8 disp(ns, 'Null space=');
9 disp(A(1,:)*ns, 'A(1,:)*ns=');
10 disp(A(2,:)*ns, 'A(2,:)*ns=');
11 disp(A(3,:)*ns, 'A(3,:)*ns=');
12 disp('This shows that the null space of A is
    orthogonal to the row space.');
```

Scilab code Exa 3.2.1 Projections onto a line

```
1 //page 155
2 clear;
3 close;
4 clc;
5 b=[1;2;3];
6 disp(b, 'b=');
7 a=[1;1;1];
8 disp(a, 'a=')
9 x=(a'*b)/(a'*a)
```

```

10 disp(x*a, 'Projection p of b onto the line through a
    is  $x^*a=$ ');
11 disp((a'*b)/(sqrt(a'*a)*sqrt(b'*b)), 'cos(theta)=');
12 //end

```

Scilab code Exa 3.2.2 Projections onto a line

```

1 //page 156
2 clear;
3 close;
4 clc;
5 a=[1;1;1];
6 disp(a, 'a=');
7 P=(a*a')/(a'*a);
8 disp(P, 'Matrix that projects onto a line through a
    =(1,1,1) is ');
9 //end

```

Scilab code Exa 3.2.3 Projections onto a line

```

1 //page 156
2 clear;
3 close;
4 clc;
5 theta=45; //Taking some value for theta
6 a=[cos(theta);sin(theta)];
7 disp(a, 'a=');
8 P=(a*a')/(a'*a);
9 disp(P, 'Projection of line onto the theta-direction
    (theta taken as 45) in the x-y plane passing
    through a is ');
10 //end

```

Scilab code Exa 3.3.1 Projection matrices

```
1 //page 165
2 clear;
3 close;
4 clc;
5 A=rand(4,4);
6 disp(A, 'A=');
7 P=A*inv(A'*A)*A';
8 disp('P=A*inv(A'*A)*A');
9 disp(P, 'Projection of a invertible 4x4 matrix on to
    the whole space is:');
10 disp('Its identity matrix.')
```

```
11 //end
```

Scilab code Exa 3.3.2 Least squares fitting of data

```
1 //page 166
2 clear;
3 close;
4 clc;
5 disp('b=C+Dt');
6 disp('Ax=b');
7 A=[1 -1;1 1;1 2];
8 disp(A, 'A=');
9 b=[1;1;3];
10 disp(b, 'b=');
11 disp('If Ax=b could be solved then they would be no
    errors, they can't be solved because the points
    are not on a line. Therefore they are solved by
    least squares.')
```

```
12 disp('so, A''Ax^=A''b');
```

```

13 x=zeros(1,2);
14 x=(A'*A)\(A'*b);
15 disp(x(1,1), 'C^ =');
16 disp(x(2,1), 'D^=');
17 disp('The best line is 9/7+4/7t')
18 //end

```

Scilab code Exa 3.4.1 Orthogonal matrices

```

1 //page 175
2 clear;
3 close;
4 clc;
5 theta=45;//Taking some value for theta.
6 Q=[cos(theta) -sin(theta);sin(theta) cos(theta)
    ];
7 disp(Q, 'Q=');
8 disp(Q', 'Q''=inv(Q)=');
9 disp('Q rotates every vector through an angle theta
    , and Q'' rotates it back through -theta.The
    columns are clearly orthogonal and they are
    orthonormal because sin^2(theta)+cos^2(theta)=1.
    ');
10 //end

```

Scilab code Exa 3.4.2 Orthogonal matrices

```

1 //page 175
2 clear;
3 close;
4 clc;
5 disp('Any permutation matrix is an orthogonal matrix
    .The columns are certainly unit vectors and

```

```

        certainly orthogonal—because the 1 appears in a
        differnt place in each column')
6 P=[0 1 0;0 0 1;1 0 0];
7 disp(P, 'P=');
8 disp(P', 'inv(P)=P''=');
9 disp(P'*P, 'And,P''*P=');
10 //end

```

Scilab code Exa 3.4.3 Projection onto a plane

```

1 //page 175
2 clear;
3 close;
4 clc;
5 disp('If we project b=(x,y,z) onto the x-y plane
        then its projection is p=(x,y,0),and is the sum
        of projection onto x- any y-axes.')
6 b=rand(3,1);
7 q1=[1;0;0];
8 disp(q1, 'q1=');
9 q2=[0;1;0];
10 disp(q2, 'q2=');
11 P=q1*q1'+q2*q2';
12 disp(P, 'Overall projection matrix ,P=');
13 disp('and,P[x;y;z]=[x;y;0] ')
14 disp('Projection onto a plane=sum of projections
        onto orthonormal q1 and q2.')
15 //end

```

Scilab code Exa 3.4.4 Least squares fitting of data

```

1 //page 166
2 clear;

```



```

3 close;
4 clc;
5 disp('y=C+Dt');
6 disp('Ax=b');
7 A=[1 -3;1 0;1 3];
8 disp(A, 'A=');
9 y=rand(3,1);
10 disp(y, 'y=');
11 disp('the columns of A are orthogonal,so')
12 x=zeros(1,2);
13 disp('([1 1 1]*y)/(A(:,1)'+A(:,1)), 'C^ =');
14 disp('([-3 0 3]*y)/(A(:,2)'+A(:,2)), 'D^ =')
15 disp('C^ gives the besy fit ny horizontal line ,
        whereas D^t is the best fit by a straight line
        through the origin.')
16 //end

```

Scilab code Exa 3.4.5 Gram Schmidt process

```

1 //page 166
2 clear;
3 close;
4 clc;
5 A=[1 0 1;1 0 0;2 1 0];//independent vectors stored
    in columns of A
6 disp(A, 'A=');
7 [m,n]=size(A);
8 for k=1:n
9     V(:,k)=A(:,k);
10    for j=1:k-1
11        R(j,k)=V(:,j)'+A(:,k);
12        V(:,k)=V(:,k)-R(j,k)*V(:,j);
13    end
14    R(k,k)=norm(V(:,k));
15    V(:,k)=V(:,k)/R(k,k);

```

```
16 end
17 disp(V, 'Q=')
```

Chapter 4

Determinants

Scilab code Exa 4.3.1 Determinant of a matrix is the product of its pivots

```
1 clear;
2 close;
3 clc;
4 n=input('Enter the value of n:');
5 for i=1
6     for j=i;
7         a(i,j)=2;
8         a(i,j+1)=-1;
9     end
10 end
11 for i=2:n-1
12     for j=i
13         a(i,j-1)=-1;
14         a(i,j)=2;
15         a(i,j+1)=-1;
16     end
17 end
18 for i=n
19     for j=i
20         a(i,j-1)=-1;
21         a(i,j)=2;
```

```

22     end
23 end
24 disp(a, 'a=');
25 [L,D,U]=lu(a)
26 determinant=1;
27 for i=1:n
28     determinant=determinant*D(i,i);
29 end
30 disp(determinant, 'Determinant=')
31 //end

```

Scilab code Exa 4.3.2 Calculation of determinant of a matrix by using cofactors

```

1 clear;
2 close;
3 clc;
4 disp('For a 3*3 matrix:');
5 disp('det A=a11(a22a33-a23a32)+a12(a23a31-a21a33)+
      a13(a21a32-a22a31)');
6 //end

```

Scilab code Exa 4.3.3 Calculation of determinant of a matrix by using cofactors

```

1 //page 214
2 clear;
3 close;
4 clc;
5 A=[2 -1 0 0;-1 2 -1 0;0 -1 2 -1;0 0 -1 2];
6 disp(A, 'A=');
7 [m,n]=size(A)
8 a=A(1,:);
9 c=[];
10 for l=1:4

```

```

11     B=A([1:0,2:4],[1:1-1,1+1:4]);
12     c11=(-1)^(1+1)*det(B);
13     c=[c;c11];
14 end
15 d=a*c;
16 disp(d)

```

Scilab code Exa 4.4.1 Inverse of a sum matrix is a difference matrix

```

1 //282
2 clear;
3 close;
4 clc;
5 A=[1 1 1;0 1 1;0 0 1];
6 disp(A,'A=');
7 n=size(A,1); d=1:n-1;
8 B=zeros(n); AA=[A,A;A,A]';
9 for j=1:n
10     for k=1:n
11         B(j,k)=det(AA(j+d,k+d));
12     end
13 end
14 disp(B,'Adjoint of A:');
15 disp(B/det(A),'inv(A):');
16 //end

```

Scilab code Exa 4.4.2 Cramers rule

```

1 //page 222
2 clear;
3 close;
4 clc;
5 //x1+3x2=0

```

```
6 //2x1+4x2=6
7 A=[1 3;2 4];
8 b=[0;6];
9 X1=[0 3;6 4];
10 X2=[1 0;2 6];
11 disp(det(X1)/det(A), 'x1=');
12 disp(det(X2)/det(A), 'x2=');
13 //end
```

Chapter 5

Eigenvalues and Eigenvectors

Scilab code Exa 5.1.1 Eigenvalues and eigenvectors

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[3 0;0 2];
6 eig=spec(A);
7 [V,Val]=spec(A);
8 disp(eig,'Eigen values:');
9 x1=V(:,1);
10 x2=V(:,2);
11 disp(x1,x2,'Eigen vectors:');
12 //end
```

Scilab code Exa 5.1.2 Eigenvalues and eigenvectors

```
1 //page 238
2 clear;
3 close;
```

```

4 clc;
5 disp('The eigen values of a projection matrix are 1
      or 0. ');
6 P=[1/2 1/2;1/2 1/2];
7 eig=spec(P);
8 [V,Val]=spec(P);
9 disp(eig, 'Eigen values: ');
10 x1=V(:,1);
11 x2=V(:,2);
12 disp(x1,x2, 'Eigen vectors: ');
13 //end

```

Scilab code Exa 5.2.1 Diagonalization

```

1 //page 238
2 clear;
3 close;
4 clc;
5 A=[1/2 1/2;1/2 1/2];
6 [V,Val]=spec(A);
7 disp(Val, 'Eigenvalue matrix: ');
8 disp(V, 'S=');
9 disp(A*V, 'AS=S*eigenvaluematrix ');
10 disp('Therefore  $\text{inv}(S)*A*S=\text{eigenvalue matrix}$  ');
11 //end

```

Scilab code Exa 5.2.2 Diagonalization

```

1 //page 238
2 clear;
3 close;
4 clc;

```



```

5 disp('The eigenvalues themselves are not so clear
      for a rotation.')
```

```

6 disp('90 degree rotation')
```

```

7 K=[0 -1;1 0];
```

```

8 disp(K, 'K=')
```

```

9 eig=spec(K);
```

```

10 [V,Val]=spec(K);
```

```

11 disp(eig, 'Eigen values:')
```

```

12 x1=V(:,1);
```

```

13 x2=V(:,2);
```

```

14 disp(x1,x2, 'Eigen vectors:');
```

```

15 //end
```

Scilab code Exa 5.2.3 Powers and Products

```

1 //page 249
```

```

2 clear;
```

```

3 close;
```

```

4 clc;
```

```

5 disp('K is rotation through 90 degree ,then K^2 is
      rotation through 180 degree and inv(k is rotation
      through -90 degree)')
```

```

6 K=[0 -1;1 0];
```

```

7 disp(K, 'K=')
```

```

8 disp(K*K, 'K^2=')
```

```

9 disp(K*K*K, 'K^3=')
```

```

10 disp(K^4, 'K^4=')
```

```

11 [V,D]=spec(K);
```

```

12 disp('K^4 is a complete rotation through 360 degree.
      ')
13 disp(D, 'Eigen value matrix ,D of K: ');
```

```

14 disp(D^4, 'and also D^4=')
```

```

15 //end
```

Scilab code Exa 5.3.1 Difference equations

```
1 //page 249
2 clear;
3 close;
4 clc;
5 A=[0 4;0 1/2];
6 disp(A, 'A=');
7 eig=spec(A);
8 disp(eig, 'Eigen values:');
9 [v,D]=spec(A);
10 u0=[v(:,1)]; //Taking u0 as the 1st eigen Vector.
11 for k=0:5
12     disp(k, 'k=');
13     u=A*u0;
14     disp(u, 'U(k+1)(K from 0 to 5)')
15     u0=u;
16 end
17 u0=[v(:,2)]; //Taking u0 as the 2nd eigen vector.
18 for k=0:5
19     disp(k, 'k=');
20     u=A*u0;
21     disp(u, 'U(k+1)=')
22     u0=u;
23 end
```

Scilab code Exa 5.5.1 Complex matrices

```
1 //page282
2 clear;
3 close;
4 clc;
```

```
5 i=sqrt(-1);
6 x=3+4*i;
7 disp(x, 'x=');
8 x_=conj(x);
9 disp(x*x_, 'xx_');
10 r=sqrt(x*x_);
11 disp(r, 'r=')
12 //end
```

Scilab code Exa 5.5.2 Inner product of a complex matrix

```
1 //282
2 clear;
3 close;
4 clc;
5 i=sqrt(-1);
6 x=[1 i]';
7 y=[2+1*i 2-4*i]';
8 disp(x'*x, 'Length of x squared:');
9 disp(y'*y, 'Length of y squared:');
10 //end
```

Chapter 6

Positive Definite Matrices

Scilab code Exa 6.1.1 Definite versus indefinite

```
1 //313
2 clear;
3 close;
4 clc;
5 disp('f(x,y)=x^2-10*x*y+y^2');
6 a=1;
7 c=1;
8 deff(' [f]=f(x,y)', 'f=x^2-10*x*y+y^2');
9 disp(f(1,1), 'f(1,1)=');
10 disp('The conditions a>0 and c>0 ensure that f(x,y)
      is positive on the x and y axes. But this
      function is negative on the line x=y, because b
      =-10 overwhelms a and c. ');
11 //end
```

Scilab code Exa 6.1.3 Maxima Minima And Saddle points

```
1 //315
```

```

2 clear;
3 close;
4 clc;
5 disp('f(x,y)=2*x^2+4*x*y+y^2');
6 A=[2 2;2 1];
7 a=1;
8 c=1;
9 b=2;
10 disp(a*c,'ac=');
11 disp(b^2,'b^2=');
12 disp('Saddle point ,as ac<b^2');

```

Scilab code Exa 6.1.4 Maxima Minima And Saddle points

```

1 //315
2 clear;
3 close;
4 clc;
5 disp('f(x,y)=2*x^2+4*x*y+y^2');
6 A=[2 2;2 1];
7 a=0;
8 c=0;
9 b=1;
10 disp(a*c,'ac=');
11 disp(b^2,'b^2=');
12 disp('Saddle point ,as ac<b^2');

```

Scilab code Exa 6.2.2 Maxima Minima And Saddle points

```

1 //313
2 clear;
3 close;
4 clc;

```

```

5 disp('f(x,y)=x^2+4*x*y+y^2');
6 a=1;
7 c=1;
8 deff('[f]=f(x,y)', 'f=x^2+4*x*y+y^2');
9 disp(f(0,0), 'f(0,0)=')
10 disp('Here 2b=4 it still does not ensure a minimum
        ,the sign of b is of no importance. Neither F nor
        f has a minimum at(0,0) because f(1,-1)=-1.')
11 //end

```

Scilab code Exa 6.3.1 Singular value decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[-1 2 2]';
6 disp(A, 'A=');
7 [U diagnol V]=svd(A);
8 disp(U, 'U=');
9 disp(diagnol, 'diagnol=');
10 disp(V', 'V''=');
11 disp(U*diagnol*V', 'A=U*diagnol*V''')
12 //end

```

Scilab code Exa 6.3.2 Singular value decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[-1 1 0;0 -1 1];
6 disp(A, 'A=');

```

```

7 [U diagnl V]=svd(A);
8 disp(U, 'U=');
9 disp(diagnl, 'Diagonal=');
10 disp(V', 'V''=');
11 disp(U*diagnl*V', 'A=U*diagonal*V''=')
12 //end

```

Scilab code Exa 6.3.3 Polar decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[1 -2;3 -1];
6 disp(A, 'A=');
7 [U S V]=svd(A);
8 Q=U*V';
9 S=V*S*V';
10 disp(Q, 'Q=');
11 disp(S, 'S=');
12 disp(Q*S, 'A=SQ=')
13 //end

```

Scilab code Exa 6.3.4 Reverse polar decomposition

```

1 //332
2 clear;
3 close;
4 clc;
5 A=[1 -2;3 -1];
6 disp(A, 'A=');
7 [U diag1 V]=svd(A);
8 Q=U*V';

```

```
9 S=[2 1;1 3];
10 disp(Q, 'Q=');
11 disp(S, 'S=')
12 disp(S'*Q, 'A=S''Q=')
13 //end
```

Chapter 7

Computations with Matrices

Scilab code Exa 7.4.1 Jacobi Method

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[2 -1;-1 2];
6 S=[2 0;0 2];
7 T=[0 1;1 0];
8 p=inv(S)*T;
9 b=[2 2]';
10 x=zeros(2,1);
11 disp(x, 'intial v & w:')
12 x_1=zeros(1,2);
13 for k=0:25
14     x_1=p*x+inv(S)*b;
15     x=x_1;
16     disp(k, 'k=')
17     disp(x_1, 'v(k+1) & w(k+1)=');
18 end
```

Scilab code Exa 7.4.2 Gauss Seidel method

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[2 -1;-1 2];
6 S=[2 0;-1 2];
7 T=[0 1;0 0];
8 b=rand(2,1);
9 p=inv(S)*T;
10 x=zeros(2,1);
11 disp(x, 'intial v & w: ')
12 x_1=zeros(1,2);
13 for k=0:25
14     x_1=p*x+inv(S)*b;
15     x=x_1;
16     disp(k, 'k=')
17     disp(x_1, 'v(k+1) & w(k+1)=');
18 end
```

Chapter 8

Linear Programming and Game Theory

Scilab code Exa 8.2.2 Minimize cx subject to x greater than or equal to zero and $Ax \leq b$

```
1 //page 238
2 clear;
3 close;
4 clc;
5 A=[1 0 1 6 2;0 1 1 0 3];
6 b=[8 9]';
7 c=[0 0 7 -1 -3]';
8 lb=[0 0 0 0 0]';
9 ub=[];
10 [x,lagr,f]=linpro(c,A,b,lb,ub);
11 disp(x,'New corner:');
12 disp(f,'Minimum cost:');
13 //end
```
