

Scilab Textbook Companion for  
Numerical Methods  
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# Book Description

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Scilab numbering policy used in this document and the relation to the above book.

**Exa** Example (Solved example)

**Eqn** Equation (Particular equation of the above book)

**AP** Appendix to Example(Scilab Code that is an Appednix to a particular Example of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means a scilab code whose theory is explained in Section 2.3 of the book.

# Contents

<b>List of Scilab Codes</b>	<b>4</b>
<b>1 Intorduction to Numerical Computing</b>	<b>5</b>
<b>3 Computer Codes and Arithmetic</b>	<b>6</b>
<b>4 Approximations and Errors in Computing</b>	<b>27</b>
<b>6 Roots of Nonlinear Equations</b>	<b>42</b>
<b>7 Direct Solutions of Linear Equations</b>	<b>56</b>
<b>8 Iterative Solution of Linear Equations</b>	<b>63</b>
<b>9 Curve Fitting Interpolation</b>	<b>68</b>
<b>10 Curve Fitting Regression</b>	<b>80</b>
<b>11 Numerical Differentiation</b>	<b>84</b>
<b>12 Numerical Integration</b>	<b>90</b>
<b>13 Numerical Solution of Ordinary Differential Equations</b>	<b>98</b>
<b>14 Boundary Value and Eigenvalue Problems</b>	<b>110</b>



# List of Scilab Codes

Exa 1.01	Theoretical Problem . . . . .	5
Exa 3.1	binary to decimal . . . . .	6
Exa 3.2	Hexadecimal to Decimal . . . . .	7
Exa 3.3	Decimal to Binary . . . . .	7
Exa 3.4	Decimal to Octal . . . . .	8
Exa 3.5	Decimal to Binary . . . . .	9
Exa 3.6	Octal to Hexadecimal . . . . .	10
Exa 3.7	Hexadecimal to Octal . . . . .	11
Exa 3.8	Binary form of negative integers . . . . .	12
Exa 3.9	16 bit word representation . . . . .	13
Exa 3.10	Floating Point Notation . . . . .	14
Exa 3.11	Integer Arithmetic . . . . .	15
Exa 3.12	Integer Arithmetic . . . . .	15
Exa 3.13	Floating Point Arithmetic Addition . . . . .	16
Exa 3.14	Floating Point Arithmetic Addition . . . . .	16
Exa 3.15	Floating Point Arithmetic Subtraction . . . . .	17
Exa 3.16	Floating Point Arithmetic Multiplication . . . . .	18
Exa 3.17	Floating Point Arithmetic division . . . . .	19
Exa 3.18	Errors in Arithmetic . . . . .	19
Exa 3.19	Errors in Arithmetic . . . . .	20
Exa 3.20	Errors in Arithmetic . . . . .	21
Exa 3.21	Errors in Arithmetic . . . . .	22
Exa 3.22	Associative law of Addition . . . . .	22
Exa 3.23	Associative law of Multiplication . . . . .	24
Exa 3.24	Distributive law of Arithmetic . . . . .	25
Exa 4.1	Greatest Precision . . . . .	27
Exa 4.2	Accuracy of Numbers . . . . .	28
Exa 4.3	Addition in Binary form . . . . .	29

Exa 4.4	Rounding off . . . . .	30
Exa 4.5	Truncation Error . . . . .	31
Exa 4.6	Truncation Error . . . . .	32
Exa 4.7	Absolute and Relative Error . . . . .	32
Exa 4.8	Machine Epsilon . . . . .	33
Exa 4.9	Propagation of Error . . . . .	33
Exa 4.10	Errors in Sequence of Computations . . . . .	34
Exa 4.11	Addition of Chain of Numbers . . . . .	35
Exa 4.12	Addition of Chain of Numbers . . . . .	36
Exa 4.13	Theoretical Problem . . . . .	37
Exa 4.14	Absolute and Relative Errors . . . . .	37
Exa 4.15	Error Evaluation . . . . .	38
Exa 4.16	Condition and Stability . . . . .	38
Exa 4.17	Theoretical Problem . . . . .	39
Exa 4.18	Difference of Square roots . . . . .	39
Exa 4.19	Theoretical Problem . . . . .	40
Exa 4.20	Theoretical Problem . . . . .	40
Exa 4.21	Induced Instability . . . . .	40
Exa 6.1	Possible initial guess values for roots . . . . .	42
Exa 6.02	Theoretical Problem . . . . .	43
Exa 6.3	Evaluating Polynomial using Horners rule . .	43
Exa 6.4	Bisection Method . . . . .	43
Exa 6.5	False Position Method . . . . .	45
Exa 6.06	Theoretical Problem . . . . .	46
Exa 6.7	Newton Raphson Method . . . . .	46
Exa 6.8	Newton Raphson Method . . . . .	47
Exa 6.9	Secant Method . . . . .	47
Exa 6.10	Theoretical Problem . . . . .	48
Exa 6.11	Fixed Point Method . . . . .	48
Exa 6.12	Fixed Point Method . . . . .	49
Exa 6.13	Fixed Point Method for non linear equations .	50
Exa 6.14	Newton Raphson Method for Non linear equations . . . . .	51
Exa 6.15	Synthetic Division . . . . .	52
Exa 6.16	Bairstow Method for Factor of polynomial . .	52
Exa 6.17	Mullers Method for Leonards equation . . . .	53
Exa 7.1	Elimination Process . . . . .	56
Exa 7.2	Basic Gauss Elimination . . . . .	57

Exa 7.3	Gauss Elimination using Partial Pivoting . . . . .	57
Exa 7.4	Gauss Jordan Elimination . . . . .	58
Exa 7.5	DoLittle LU Decomposition . . . . .	59
Exa 7.6	Choleskys Factorisation . . . . .	61
Exa 7.7	Ill Conditioned Systems . . . . .	62
Exa 8.1	Gauss Jacobi Iteration Method . . . . .	63
Exa 8.2	Gauss Seidel Iterative Method . . . . .	64
Exa 8.3	Gauss Seidel Iterative Method . . . . .	65
Exa 8.4	Gauss Seidel Iterative Method . . . . .	66
Exa 9.1	Polynomial Forms . . . . .	68
Exa 9.2	Shifted Power form . . . . .	68
Exa 9.3	Linear Interpolation . . . . .	69
Exa 9.4	Lagrange Interpolation . . . . .	70
Exa 9.5	Lagrange Interpolation . . . . .	71
Exa 9.6	Newton Interpolation . . . . .	72
Exa 9.7	Newton Divided Difference Interpolation . . . . .	73
Exa 9.8	Newton Gregory Forward Difference Formula . . . . .	74
Exa 9.9	Newton Backward Difference Formula . . . . .	75
Exa 9.10	Splines . . . . .	75
Exa 9.11	Cubic Spline Interpolation . . . . .	77
Exa 9.12	Cubic Spline Interpolation . . . . .	78
Exa 10.1	Fitting a Straight line . . . . .	80
Exa 10.2	Fitting a Power Function Model to given data . . . . .	80
Exa 10.3	Fitting a Straight line using Regression . . . . .	81
Exa 10.4	Curve Fitting . . . . .	82
Exa 10.5	Plane Fitting . . . . .	83
Exa 11.1	First order Forward Difference . . . . .	84
Exa 11.2	Three Point Formula . . . . .	84
Exa 11.3	Error Analysis . . . . .	85
Exa 11.4	Approximate Second Derivative . . . . .	86
Exa 11.5	Differentiation of Tabulated Data . . . . .	86
Exa 11.6	Three Point Central Difference Formula . . . . .	87
Exa 11.7	Second order Derivative . . . . .	88
Exa 11.8	Richardsons Extrapolation Technique . . . . .	89
Exa 12.1	Trapezoidal Rule . . . . .	90
Exa 12.2	Trapezoidal Rule . . . . .	91
Exa 12.3	Simpsons 1 by 3 rule . . . . .	92
Exa 12.4	Simpsons 1 by 3 rule . . . . .	93

Exa 12.5	Simpsons 3 by 8 rule . . . . .	94
Exa 12.6	Booles Five Point Formula . . . . .	94
Exa 12.7	Romberg Integration Formula . . . . .	95
Exa 12.8	Two Point Gauss Legefre Formula . . . . .	96
Exa 12.9	Gaussian Two Point Formula . . . . .	96
Exa 12.10	Gauss Legendre Three Point Formula . . . . .	97
Exa 13.1	Taylor Method . . . . .	98
Exa 13.2	Recursive Taylor Method . . . . .	98
Exa 13.3	Picards Method . . . . .	99
Exa 13.4	Eulers Method . . . . .	100
Exa 13.5	Error Estimation in Eulers Method . . . . .	101
Exa 13.6	Heuns Method . . . . .	102
Exa 13.7	Polygon Method . . . . .	103
Exa 13.8	Classical Runge Kutta Method . . . . .	103
Exa 13.9	Optimum Step size . . . . .	104
Exa 13.10	Milne Simpson Predictor Corrector Method	105
Exa 13.11	Adams Bashforth Moulton Method . . . . .	106
Exa 13.12	Milne Simpson Method Using Modifier . . .	107
Exa 13.13	System of Differential Equations . . . . .	107
Exa 13.14	Higher Order Differential Equations . . . . .	108
Exa 14.1	Shooting Method . . . . .	110
Exa 14.2	Finite Difference Method . . . . .	111
Exa 14.3	Eigen Vectors . . . . .	112
Exa 14.4	Fadeev Leverrier Method . . . . .	113
Exa 14.5	Eigen Vectors . . . . .	113
Exa 14.6	Power Method . . . . .	114
Exa 15.1	Elliptic Equations . . . . .	115
Exa 15.2	Liebmans Iterative Method . . . . .	116
Exa 15.3	Poissons Equation . . . . .	117
Exa 15.4	Gauss Siedel Iteration . . . . .	118
Exa 15.5	Initial Value Problems . . . . .	118
Exa 15.6	Crank Nicholson Implicit Method . . . . .	119
Exa 15.7	Hyperbolic Equations . . . . .	120

# Chapter 1

## Intorduction to Numerical Computing

Scilab code Exa 1.01 Theoritical Problem

```
1 //Example No. 1_01
2 //Pg No. 6
3 disp('Theoritical Problem')
4 disp('For Details go to page no. 6')
```

---

# Chapter 3

## Computer Codes and Arithmetic

Scilab code Exa 3.1 binary to decimal

```
1 //Example No. 3_01
2 //Binary to decimal
3 //Pg No. 45
4 clear ;close ; clc ;
5
6 b = '1101.1101'
7 v = strsplt(b,'.')      // splitting integral part and
                           fraction part
8 integralp = str2code(v(1)) //converting strings to
                           numbers
9 fractionp = str2code(v(2))
10 li = length(integralp)   // lenght of integral part
11 lf = length(fractionp)   // and fractional part
12 di = 0 ;// Initializing integral part and decimal
           part
13 df = 0 ;
14 for i = 1:li
15     di = 2*di+integralp(i)
16 end
```

```
17 for i = lf:-1:1
18     df = df/2 + fractionp(i)
19 end
20 df = df/2 ;
21 d = di + df ; //Integral and fractional parts
22 disp(d, 'Decimal value = ')
```

---

### Scilab code Exa 3.2 Hexadecimal to Decimal

```
1 //Example No. 3_02
2 //hexadecimal to decimal
3 //Pg No. 46
4 clear ; close ; clc ;
5
6 h = '12AF' ;
7 u = str2code(h)
8 u = abs(u)
9 n = length(u)
10 d = 0
11 for i = 1:n
12     d = d*16 + u(i)
13 end
14 disp(d, 'Decimal value = ')
15 //Using Scilab Function
16 d = hex2dec(h)
17 disp(d, 'Using scilab function Decimal value = ')
```

---

### Scilab code Exa 3.3 Decimal to Binary

```
1 //Example No. 3_03
2 //Decimal to Binary
3 //Pg No. 47
4 clear; close ; clc;
```

```

5
6 d = 43.375 ;
7 //Separating integral part and fractional parts
8 dint = floor(d)
9 dfrac = d - dint
10
11 //Integral Part
12 i = 1 ;
13 intp = dec2bin(dint)
14
15 //Fractional part
16 j = 1 ;
17 while dfrac ~= 0
18     fracp(j) = floor(dfrac*2)
19     dfrac = dfrac*2 - floor(dfrac*2)
20     j = j+1 ;
21 end
22 fracp = strcat(string(fracp))
23
24 b = strcat([intp,fracp],'.') //combining integral
    part and fractional part
25 disp(b,'Binary equivalent = ')

```

---

### Scilab code Exa 3.4 Decimal to Octal

```

1 //Example No. 3_04
2 //Decimal to Octal
3 //Pg No. 48
4 clear ; close ; clc ;
5
6 d = 163 ;
7 oct = dec2oct(d)
8 disp(oct,'Octal number = ')

```

---

### Scilab code Exa 3.5 Decimal to Binary

```
1 //Example No. 3_05
2 //Decimal to binary
3 //Pg No. 48
4 clear ; close ; clc ;
5
6 d = 0.65
7 j = 1 ;
8
9 while d ~= 0
10    fracp(j) = floor(d*2) //integral part of d*2
11    d = d*2 - floor(d*2) //Fractional part of d*2
12    j = j+1 ;
13    decp(j-1) = d
14    p = 1
15
16    for i = 1:j-2
17        if abs(d - decp(i))< 0.001 then //Condition
           for terminating the recurring binary
           equivalent by
18            p = 0                                //finding
               out if the new fractional part is
               equal to any of the previous
               fractonal parts
19            break
20        end
21    end
22
23    if p == 0 then
24        break
25    end
26
27 end
```

```

28 rec_p = frACP(i+1:j-1)      // Recurring part
29
30 rec_p = strcat(string(rec_p))
31 frACP = strcat(string(frACP))
32
33 disp(strcat([frACP,rec_p]),'Binary equivalent = ')

```

---

### Scilab code Exa 3.6 Octal to Hexadecimal

```

1 //Example No. 3_06
2 //Octal to Hexadecimal
3 //Pg No. 49
4 clear ; close ; clc ;
5
6 oct = '243' ;
7 u = str2code(oct)
8 n = length(u)
9 for i = 1:n
10    b(i) = dec2bin(u(i)) //Converting each digit to
        binary equivalent
11    if length(b(i)) == 2 then           //making the
        binary equivalents into a groups of triplets
12        b(i) = strcat(['0',b(i)])
13    elseif length(b(i)) == 1
14        b(i) = strcat(['0','0',b(i)])
15    end
16 end
17 bin = strcat(b) //combining all the triplets
18 i = 1 ;
19 while length(bin) > 4
20    OtoH = strspli(bin,length(bin)-4) //splitting
        the binary equivalent into groups of binary
        quadruplets
21    bin = OtoH(1)
22    h(i) = OtoH(2)

```

```

23     i = i+1
24 end
25 h(i) = bin ;
26 h = h($:-1:1)
27 h = bin2dec(h)
28 h = dec2hex(h)
29 h = strcat(h)
30
31 disp(h, 'Hexadecimal equivalent of octal number 243
    is ')

```

---

### Scilab code Exa 3.7 Hexadecimal to Octal

```

1 //Example No. 3_07
2 //Hexadecimal to Octal
3 //Pg No. 49
4 clear ; close ; clc ;
5
6 h = '39.B8' ;
7 h = strsplt(h, '.') //separating integral part and
    fractional part
8 cint = abs(str2code(h(1)))
9 cfrac = abs(str2code(h(2)))
10 bint = dec2bin(cint)
11 bfrac = dec2bin(cfrac)
12 bint = strcat(bint)
13 bfrac = strcat(bfrac)
14
15 //Integral Part
16 i = 1 ;
17 while length(bint) > 3
18     HtoO = strsplt(bint, length(bint)-3)
19     bint = HtoO(1)
20     oint(i) = HtoO(2)
21     i = i+1 ;

```

```

22 end
23 oint(i) = bint
24 oint = oint($:-1:1)
25 oint = bin2dec(oint)
26
27 //Fraction Part
28 i = 1 ;
29 while length(bfrac)> 3
30     Hto0 = strsplit(bfrac,3)
31     bfrac = Hto0(2)
32     ofrac(i) = Hto0(1)
33     i = i+1
34 end
35 ofrac(i) = bfrac
36 ofrac = bin2dec(ofrac)
37
38 //Combining integral part and fraction part
39 oct = strcat([strcat(string(oint)),strcat(string(
    ofrac))],'.')
40 disp(oct,'Octal number equivalent of Hexadecimal
    number 39.B8 is ')

```

---

### Scilab code Exa 3.8 Binary form of negative integers

```

1 //Example No. 3_08
2 //–ve Integer to binary
3 //Pg No. 50
4 clear ; close ; clc ;
5
6 negint = -13
7 posbin = dec2bin(abs(negint))
8 posbin = strcat(['0',posbin])
9 compl_1 = strsubst(posbin,'0','d')
10 compl_1 = strsubst(compl_1,'1','0')
11 compl_1 = strsubst(compl_1,'d','1')

```

```
12 compl_2 = dec2bin(bin2dec(compl_1) + 1)
13
14 disp(compl_2, 'Binary equivalent of -13 is ')
```

---

### Scilab code Exa 3.9 16 bit word representation

```
1 //Example No. 3_09
2 //Binary representation
3 //Pg No. 51
4 clear ; close ; clc ;
5
6 n = -32768
7 compl_32767 = dec2bin(bitcmp(abs(n)-1,16) + 1)
8 disp(compl_32767, 'binary equivalent of -32767 is ')
9
10 n_1 = -1
11 dcomp = bitcmp(1,16)
12 compl_1 = dec2bin(dcomp+1)
13 disp(compl_1, 'binary equivalent of -1 is ')
14 compl_32767_code = str2code(compl_32767)
15 compl_1_code = str2code(compl_1)
16 summ(1) = 1 //since -32768 is a negative number
17 c = 0
18 for i = 16:-1:2
19     summ(i) = compl_32767_code(i) + compl_1_code(i) +
                  c
20     if summ(i) == 2 then
21         summ(i) = 0
22         c = 1
23     else
24         c = 0
25     end
26 end
27 binfinal = strcat(string(summ))
28 disp(binfinal, 'Binary equivalent of -32768 in a 16
```

bit word is ')

---

### Scilab code Exa 3.10 Floating Point Notation

```
1 //Example No. 3_10
2 //Floating Point Notation
3 //Pg No. 52
4 clear ; close ; clc ;
5
6 function [m,e] =float_notation(n)
7 m = n ;
8 for i = 1:16
9     if abs(m) >= 1 then
10        m = n/10^i
11        e = i
12    elseif abs(m) < 0.1
13        m = n*10^i
14        e = -i
15    else
16        if i == 1 then
17            e = 0
18        end
19        break ;
20    end
21 end
22 endfunction
23
24 [m,e] = float_notation(0.00596)
25 mprintf ('\n 0.00596 is expressed as %f*10^%i \n',m,
26 e)
26 [m,e] = float_notation(65.7452)
27 mprintf ('\n 65.7452 is expressed as %f*10^%i \n',m,
28 e)
28 [m,e] = float_notation(-486.8)
29 mprintf ('\n -486.8 is expressed as %f*10^%i \n',m,e)
```

)

---

### Scilab code Exa 3.11 Integer Arithmetic

```
1 //Example No. 3_11
2 //Integer Arithmetic
3 //Pg No. 53
4 clear ;close ;clc ;
5
6 disp(int(25 + 12))
7 disp(int(25 - 12))
8 disp(int(12 - 25))
9 disp(int(25*12))
10 disp(int(25/12))
11 disp(int(12/25))
```

---

### Scilab code Exa 3.12 Integer Arithmetic

```
1 //Example No. 3_12
2 //Integer Arithmetic
3 //Pg No. 53
4 clear ;close ;clc ;
5 a = 5 ;
6 b = 7 ;
7 c = 3 ;
8 Lhs = int((a + b)/c)
9 Rhs = int(a/c) + int(b/c)
10 disp(Rhs , 'a/c + b/c = ' , Lhs , '(a+b)/c = ')
11 if Lhs ~= Rhs then
12     disp('The results are not identical. This is
           because the remainder of an integer division
           is always truncated')
13 end
```

---

### Scilab code Exa 3.13 Floating Point Arithmetic Addition

```
1 //Example No. 3_13
2 //Floating Point Arithmetic
3 //Pg No. 54
4 clear ;close ;clc ;
5
6 fx = 0.586351 ;
7 Ex = 5 ;
8 fy = 0.964572 ;
9 Ey = 2 ;
10 [Ez,n] = max(Ex,Ey)
11 if n == 1 then
12     fy = fy*10^(Ey-Ex)
13     fz = fx + fy
14     if fz > 1 then
15         fz = fz*10^(-1)
16         Ez = Ez + 1
17     end
18     disp(fz,'fz = ',fy,'fy = ',Ez,'Ez = ')
19 else
20     fx = fx*10^(Ex - Ey)
21     fz = fx + fy
22     if fz > 1 then
23         fz = fz*10^(-1)
24         Ez = Ez + 1
25     end
26     disp(fz,'fz = ',fx,'fx = ',Ez,'Ez = ')
27 end
28 mprintf('\n z = %f E%i \n',fz,Ez)
```

---

### Scilab code Exa 3.14 Floating Point Arithmetic Addition

```

1 //Example No. 3_14
2 //Floating Point Arithmetic
3 //Pg No. 54
4 clear ;close ;clc ;
5
6 fx = 0.735816 ;
7 Ex = 4 ;
8 fy = 0.635742 ;
9 Ey = 4 ;
10 [Ez,n] = max(Ex,Ey)
11 if n == 1 then
12     fy = fy*10^(Ey-Ex)
13     fz = fx + fy
14     if fz > 1 then
15         fz = fz*10^(-1)
16         Ez = Ez + 1
17     end
18     disp(fz,'fz = ',fy,'fy = ',Ez,'Ez = ')
19 else
20     fx = fx*10^(Ex - Ey)
21     fz = fx + fy
22     if fz > 1 then
23         fz = fz*10^(-1)
24         Ez = Ez + 1
25     end
26     disp(fz,'fz = ',fx,'fx = ',Ez,'Ez = ')
27 end
28 mprintf('\n z = %f E%i \n',fz,Ez)

```

---

### Scilab code Exa 3.15 Floating Point Arithmetic Subtraction

```

1 //Example No. 3_15
2 //Floating Point Arithmetic
3 //Pg No. 54
4 clear ;close ;clc ;

```

```

5
6 fx = 0.999658 ;
7 Ex = -3 ;
8 fy = 0.994576 ;
9 Ey = -3 ;
10 Ez = max(Ex,Ey)
11 fz = fy*10^(Ey-Ex)
12 fz = fx - fy
13 disp(fz,'fz = ',Ez,'Ez = ')
14 mprintf('\n z = %f E%i \n',fz,Ez)
15 if fz < 0.1 then
16     fz = fz*10^6           // Since we are using 6
                           significant digits
17     n = length(string(fz))
18     fz = fz/10^n
19     Ez = Ez + n - 6
20     mprintf('\n z = %f E%i (normalised) \n',fz,Ez)
21 end

```

---

### Scilab code Exa 3.16 Floating Point Arithmetic Multiplication

```

1 //Example No. 3_16
2 //Floating Point Arithmetic
3 //Pg No. 55
4 clear ;close ;clc ;
5
6 fx = 0.200000 ;
7 Ex = 4 ;
8 fy = 0.400000 ;
9 Ey = -2 ;
10 fz = fx*fy
11 Ez = Ex + Ey
12 mprintf('\n fz = %f \n Ez = %i \n z = %f E%i \n',fz,
          Ez,fz,Ez)
13 if fz < 0.1 then

```

```

14     fz = fz*10
15     Ez = Ez - 1
16     mprintf( '\n z = %f E%i (normalised) \n', fz, Ez)
17 end

```

---

### Scilab code Exa 3.17 Floating Point Arithmetic division

```

1 //Example No. 3_17
2 //Floating Point Arithmetic
3 //Pg No. 55
4 clear ;close ;clc ;
5
6 fx = 0.876543 ;
7 Ex = -5 ;
8 fy = 0.200000 ;
9 Ey = -3 ;
10 fz = fx/fy
11 Ez = Ex - Ey
12 mprintf( '\n fz = %f \n Ez = %i \n z = %f E%i \n', fz,
           Ez, fz, Ez)
13
14 if fz > 1 then
15     fz = fz/10
16     Ez = Ez + 1
17     mprintf( '\n z = %f E%i (normalised) \n', fz, Ez)
18 end

```

---

### Scilab code Exa 3.18 Errors in Arithmetic

```

1 //Example No. 3_18
2 //Floating Point Arithmetic
3 //Pg No. 56
4 clear ;close ;clc ;

```

```

5
6 fx = 0.500000 ;
7 Ex = 1 ;
8 fy = 0.100000 ;
9 Ey = -7 ;
10 [Ez ,n] = max(Ex ,Ey)
11 if n == 1 then
12     fy = fy*10^(Ey-Ex)
13     fz = fx + fy
14     if fz > 1 then
15         fz = fz*10^(-1)
16         Ez = Ez + 1
17     end
18     disp(fy , 'fy = ' ,Ez , 'Ez = ')
19 else
20     fx = fx*10^(Ex - Ey)
21     fz = fx + fy
22     if fz > 1 then
23         fz = fz*10^(-1)
24         Ez = Ez + 1
25     end
26     disp(fx , 'fx = ' ,Ez , 'Ez = ')
27 end
28 mprintf ('\n fz = %f \n z = %f E%i \n ',fz ,fz ,Ez)

```

---

### Scilab code Exa 3.19 Errors in Arithmetic

```

1 //Example No. 3_19
2 //Floating Point Arithmetic
3 //Pg No. 56
4 clear ;close ;clc ;
5
6 fx = 0.350000 ;
7 Ex = 40 ;
8 fy = 0.500000 ;

```

```

9 Ey = 70 ;
10 fz = fx*fy
11 Ez = Ex + Ey
12 mprintf( '\n fz = %f \n Ez = %i \n z = %f E%i \n ', fz ,
Ez , fz , Ez )
13 if fz < 0.1 then
14     fz = fz*10
15     Ez = Ez - 1
16     mprintf( '\n z = %f E%i ( normalised ) \n ', fz , Ez )
17 end

```

---

### Scilab code Exa 3.20 Errors in Arithmetic

```

1 //Example No. 3_20
2 //Floating Point Arithmetic
3 //Pg No. 56
4 clear ; close ; clc ;
5
6 fx = 0.875000 ;
7 Ex = -18 ;
8 fy = 0.200000 ;
9 Ey = 95 ;
10 fz = fx/fy
11 Ez = Ex - Ey
12 mprintf( '\n fz = %f \n Ez = %i \n z = %f E%i \n ', fz ,
Ez , fz , Ez )
13
14 if fz > 1 then
15     fz = fz/10
16     Ez = Ez + 1
17     mprintf( '\n z = %f E%i ( normalised ) \n ', fz , Ez )
18 end

```

---

### Scilab code Exa 3.21 Errors in Arithmetic

```
1 //Example No. 3_21
2 //Floating Point Arithmetic
3 //Pg No. 57
4 clear ; close ; clc ;
5
6 fx = 0.500000 ;
7 Ex = 0 ;
8 fy = 0.499998 ;
9 Ey = 0 ;
10 Ez = 0 ;
11 fz = fx - fy
12 disp(fz, 'fz = ', Ez, 'Ez = ')
13 mprintf('\n z = %f E%i \n', fz, Ez)
14 if fz < 0.1 then
15     fz = fz*10^6
16     n = length(string(fz))
17     fz = fz/10^n
18     Ez = Ez + n - 6
19     mprintf ('\n z = %f E%i (normalised) \n', fz, Ez)
20 end
```

---

### Scilab code Exa 3.22 Associative law of Addition

```
1 //Example No. 3_22
2 //Laws of Arithmetic
3 //Pg No. 57
4 clear ; close ; clc ;
5 function [fz,Ez] = add_sub(fx,Ex,fy,Ey) //addition
    and subtraction fuction
6 if fx*fy >= 0 then
7     //Addition
8     [Ez,n] = max(Ex,Ey)
9     if n == 1 then
```

```

10      fy = fy*10^(Ey-Ex)
11      fz = fx + fy
12      if fz > 1 then
13          fz = fz*10^(-1)
14          Ez = Ez + 1
15      end
16  else
17      fx = fx*10^(Ex - Ey)
18      fz = fx + fy
19      if fz > 1 then
20          fz = fz*10^(-1)
21          Ez = Ez + 1
22      end
23  end
24
25 else
26 // Subtraction
27 [Ez,n] = max(Ex,Ey)
28 if n == 1 then
29     fy = fy*10^(Ey-Ex)
30     fz = fx + fy
31     if abs(fz) < 0.1 then
32         fz = fz*10^6
33         fz = floor(fz)
34         nfz = length(string(abs(fz)))
35         fz = fz/10^nfz
36         Ez = nfz - 6
37     end
38 else
39     fx = fx*10^(Ex - Ey)
40     fz = fx + fy
41     if fz < 0.1 then
42         fz = fz*10^6
43         fz = int(fz)
44         nfz = length(string(abs(fz)))
45         fz = fz/10^nfz
46         Ez = nfz - 6
47     end

```

```

48     end
49 end
50 endfunction
51
52 fx = 0.456732
53 Ex = -2
54 fy = 0.243451
55 Ey = 0
56 fz = -0.24800
57 Ez = 0
58
59 [fx_y,Exy] = add_sub(fx,Ex,fy,Ey)
60 [fx_y_z,Exy_z] = add_sub(fxy,Exy,fz,Ez)
61 [fy_z,Eyz] = add_sub(fy,Ey,fz,Ez)
62 [fx_y_z,Ex_yz] = add_sub(fx,Ex,fyz,Eyz)
63 mprintf('fxy = %f\n Exy = %i \n fx_y_z = %f\n Exy_z =
             %i \n fyz = %f \n Eyz = %i \n fx_y_z = %f \n
             Ex_yz = %i \n ',fxy,Exy,fx_y_z,Eyz,fxy_z,Ex_yz,
             Ex_yz)
64
65 if   fx_y_z ~= fx_y_z | Exy_z ~= Ex_yz then
66     disp('(x+y) + z ~= x + (y+z)')
67 end

```

---

### Scilab code Exa 3.23 Associative law of Multiplication

```

1 //Example No. 3_23
2 //Associative law
3 //Pg No. 58
4 clear ; close ; clc ;
5 x = 0.400000*10^40
6 y = 0.500000*10^70
7 z = 0.300000*10^(-30)
8 disp('In book they have considered the maximum
       exponent can be only 99, since 110 is greater

```

```

    than 99 the result is erroneous')
9 disp((x*y)*z, 'xy_z = ', 'but in scilab the this value
      is much larger than 110 so we get a correct
      result ')
10 disp(x*(y*z), 'x_yz = ')

```

---

### Scilab code Exa 3.24 Distributive law of Arithmetic

```

1 //Example No. 3_24
2 //Distributive law
3 //Pg No. 58
4 clear ;close ;clc ;
5
6 x = 0.400000*10^1 ;
7 fx = 0.400000
8 Ex = 1
9 y = 0.200001*10^0 ;
10 z = 0.200000*10^0 ;
11 x_yz = x*(y-z)
12 x_yz = x_yz*10^6
13 x_yz = floor(x_yz) //considering only six
      significant digits
14 n = length(string(x_yz))
15 fx_yz = x_yz/10^n
16 Ex_yz = n - 6
17 x_yz = fx_yz *10^Ex_yz
18 disp(x_yz, 'x_yz = ')
19
20 fxy = fx*y
21 fxy = fxy*10^6
22 fxy = floor(fxy) //considering only six significant
      digits
23 n = length(string(fxy))
24 fxy = fxy/10^n
25 Exy = n - 6

```

```
26 xy = fxy * 10^Exy
27
28 fxz = fx*z
29 fxz = fxz*10^6
30 fxz = floor(fxz) // considering only six significant
    digits
31 n = length(string(fxz))
32 fxz = fxz/10^n
33 Exz = n - 6
34 xz = fxz * 10^Exz
35
36 xy_xz = xy - xz
37 disp(xy_xz, 'xy_xz = ')
```

---

# Chapter 4

## Approximations and Errors in Computing

Scilab code Exa 4.1 Greatest Precision

```
1 //Example No. 4_01
2 //Greatest precision
3 //Pg No. 63
4 clear ; close ; clc ;
5
6 a = '4.3201'
7 b = '4.32'
8 c = '4.320106'
9 na = length(a)-strindex(a,'.')
10 mprintf ('\n %s has a precision of 10^-%i\n',a,na)
11 nb = length(b)-strindex(b,'.')
12 mprintf ('\n %s has a precision of 10^-%i\n',b,nb)
13 nc = length(c)-strindex(c,'.')
14 mprintf ('\n %s has a precision of 10^-%i\n',c,nc)
15 [n,e] = max(na,nb,nc)
16 if e ==1 then
17     mprintf ('\n The number with highest precision is
18 %s\n',a)
19 elseif e == 2
```

```

19     mprintf ('\n The number with highest precision is
20           %s\n',b)
21 else
22     mprintf ('\n The number with highest precision is
23           %s\n',c)
24 end

```

---

### Scilab code Exa 4.2 Accuracy of Numbers

```

1 //Example No. 4_02
2 //Accuracy of numbers
3 //Pg No. 63
4 clear ;close ;clc ;
5
6 function n = sd(x)
7     nd = strindex(x,'.') //position of point
8     num = str2code(x)
9     if isempty(nd) & num(length(x)) == 0 then
10         mprintf ('Accuracy is not specified\n')
11         n = 0 ;
12     else
13         if num(1)>= 1 & isempty(nd) then
14             n = length(x)
15         elseif num(1) >= 1 & ~isempty(nd) then
16             n = length(x) - 1
17         else
18             for i = 1:length(x)
19                 if num(i) >= 1 & num(i) <= 9 then
20                     break
21                 end
22             end
23             n = length(x)- i + 1
24         end
25     end
26 endfunction

```

```

27 a = '95.763'
28 na = sd(a)
29 mprintf('%s has %i significant digits\n',a,na)
30 b = '0.008472'
31 nb = sd(b)
32 mprintf('%s has %i significant digits. The leading or
            higher order zeros are only place holders\n',b,
            nb)
33 c = '0.0456000'
34 nc = sd(c)
35 mprintf('%s has %i significant digits\n',c,nc)
36 d = '36'
37 nd = sd(d)
38 mprintf('%s has %i significant digits\n',d,nd)
39 e = '3600'
40 se = sd(e)
41 f = '3600.00'
42 nf = sd(f)
43 mprintf('%s has %i significant digits\n',f,nf)

```

---

### Scilab code Exa 4.3 Addition in Binary form

```

1 //Example No. 4_03
2 //Pg No. 64
3 clear ; close ; clc ;
4
5 a = 0.1
6 b = 0.4
7 for i = 1:8
8     afrac(i) = floor(a*2)
9     a = a*2 - floor(a*2)
10    bfrac(i) = floor(b*2)
11    b = b*2 - floor(b*2)
12 end
13 afracs = '0' + '.' + strcat(string(afrac)) // string

```

```

        form binary equivalent of a i.e 0.1
14 bfrac_s = '0' + '.' + strcat(string(bfrac))
15 mprintf ('\n 0.1_10 = %s \n 0.4_10 = %s \n ', afracs
           , bfrac_s)
16 for j = 8:-1:1
17     summ(j) = afrac(j) + bfrac(j)
18     if summ(j) > 1 then
19         summ(j) = summ(j)-2
20         afrac(j-1) = afrac(j-1) + 1
21     end
22 end
23 summ_dec = 0
24 for k = 8:-1:1
25     summ_dec = summ_dec + summ(k)
26     summ_dec = summ_dec*1/2
27 end
28 disp(summ_dec, 'sum =')
29 disp('Note : The answer should be 0.5, but it is not
       so. This is due to the error in conversion from
       decimal to binary form.')

```

---

#### Scilab code Exa 4.4 Rounding off

```

1 //Example No. 4_04
2 //Rounding-Off
3 //Pg No. 66
4 clear ; close ; clc ;
5
6 fx = 0.7526
7 E =3
8 gx = 0.835
9 d = E - (-1)
10 //Chopping Method
11 Approx_x = fx*10^E
12 Err = gx*10^(E-d)

```

```

13 mprintf ('\n Chooping Method : \n Approximate x = %.4
           f*10^%i \n Error = %.4f \n ',fx,E,Err)
14 //Symmetric Method
15 if gx >= 0.5 then
16     Err = (gx -1)*10^(-1)
17     Approx_x = (fx + 10^(-d))*10^E
18 else
19     Approx_x = fx*10^E
20     Err = gx * 10^(E-d)
21 end
22 mprintf ('\n Symmetric Rounding :\n Approximate x = %
           .4f*10^%i \n Error = %.4f \n ',fx + 10^(-d),E,Err
)

```

---

### Scilab code Exa 4.5 Truncation Error

```

1 //Example No. 4_05
2 //Truncation Error
3 //Pg No. 68
4 clear ; close ; clc ;
5
6 x = 1/5
7 //When first three terms are used
8 Trunc_err = x^3/factorial(3) + x^4/factorial(4) + x
           ^5/factorial(5) + x^6/factorial(6)
9 mprintf ('\n a) When first three terms are used \n
           Truncation error = %.6E \n ',Trunc_err)
10
11 //When four terms are used
12 Trunc_err = x^4/factorial(4) + x^5/factorial(5) + x
           ^6/factorial(6)
13 mprintf ('\n b) When first four terms are used \n
           Truncation error = %.6E \n ',Trunc_err)
14
15 //When Five terms are used

```

```
16 Trunc_err = x^5/factorial(5) + x^6/factorial(6)
17 mprintf ('\n c) When first five terms are used \n
           Truncation error = %.6E \n ',Trunc_err)
```

---

### Scilab code Exa 4.6 Truncation Error

```
1 //Example No. 4_06
2 //Truncation Error
3 //Pg No. 68
4 clear ; close ; clc ;
5
6 x = -1/5
7 //When first three terms are used
8 Trunc_err = x^3/factorial(3) + x^4/factorial(4) + x
           ^5/factorial(5) + x^6/factorial(6)
9 mprintf ('\n a) When first three terms are used \n
           Truncation error = %.6E \n ',Trunc_err)
10
11 //When four terms are used
12 Trunc_err = x^4/factorial(4) + x^5/factorial(5) + x
           ^6/factorial(6)
13 mprintf ('\n b) When first four terms are used \n
           Truncation error = %.6E \n ',Trunc_err)
14
15 //When Five terms are used
16 Trunc_err = x^5/factorial(5) + x^6/factorial(6)
17 mprintf ('\n c) When first five terms are used \n
           Truncation error = %.6E \n ',Trunc_err)
```

---

### Scilab code Exa 4.7 Absolute and Relative Error

```
1 //Example No. 4_07
2 //Absolute and Relative Errors
```

```

3 //Pg No. 71
4 clear ; close ; clc ;
5
6 h_bu_t = 2945 ;
7 h_bu_a = 2950 ;
8 h_be_t = 30 ;
9 h_be_a = 35 ;
10 e1 = abs(h_bu_t - h_bu_a)
11 e1_r = e1/h_bu_t
12 e2 = abs(h_be_t - h_be_a)
13 e2_r = e2/h_be_t
14 mprintf('n For Building : n Absolute error , e1 =
    %i \n Relative error , e1_r = %.2f percent \n ', e1, e1_r*100)
15 mprintf('n For Beam : n Absolute error , e2 = %i \
    n Relative error , e2_r = %.2G percent \n ', e2,
    e2_r*100)

```

---

### Scilab code Exa 4.8 Machine Epsilon

```

1 //Example No. 4_08
2 //Machine Epsilon
3 //Pg No. 72
4 clear ; close ; clc ;
5
6 deff('q = Q(p)', 'q = 1 + (p-1)*log10(2) ')
7 p = 24
8 q = Q(p)
9 mprintf('q = %.1f \n We can say that the computer
    can store numbers with %i significant decimal
    digits \n ', q, q)

```

---

### Scilab code Exa 4.9 Propagation of Error

```

1 //Example No. 4_09
2 //Propagation of Error
3 //Pg No. 75
4 clear ; close ; clc ;
5
6 x = 0.1234*10^4
7 y = 0.1232*10^4
8 d = 4
9 er_x = 10^(-d + 1)/2
10 er_y = 10^(-d + 1)/2
11 ex = x*er_x
12 ey = y*er_y
13 ez = abs(ex) + abs(ey)
14 er_z = abs(ez)/abs(x-y)
15
16 mprintf (' \n | er_x | <= %.2f o/o \n | er_y | <= %.2f o/o \
n ex = %.3f \n ey = %.3f \n | ez | = %.3f \n | er_z |
= %.2f o/o \n ', er_x *100 , er_y*100 , ex , ey , ez , er_z
*100)

```

---

### Scilab code Exa 4.10 Errors in Sequence of Computations

```

1 //Example No. 4_10
2 //Errors in Sequence of Computations
3 //Pg No. 77
4 clear ; close ; clc ;
5
6 x_a = 2.35 ;
7 y_a = 6.74 ;
8 z_a = 3.45 ;
9 ex = abs(x_a)*10^(-3+1)/2
10 ey = abs(y_a)*10^(-3+1)/2
11 ez = abs(z_a)*10^(-3+1)/2
12 exy = abs(x_a)*ey + abs(y_a)*ex
13 ew = abs(exy) + abs(ez)

```

```
14 mprintf( '\n ex = %.5f \n ey = %.5f \n ez = %.5f \n
    exy = %.5f \n ew = %.5f \n ',ex,ey,ez,exy,ew)
```

---

### Scilab code Exa 4.11 Addition of Chain of Numbers

```
1 //Example No. 4_11
2 //Addition of Chain of Numbers
3 //Pg No. 77
4 clear ; close ; clc ;
5
6 x = 9678 ;
7 y = 678 ;
8 z = 78 ;
9 d = 4 ; //length of mantissa
10 fx = x/10^4
11 fy = y/10^4
12 fu = fx + fy
13 Eu = 4
14 if fu >= 1 then
15     fu = fu/10
16     Eu = Eu + 1
17 end
18 //since length of mantissa is only four we need to
   maintain only four places in decimal , so
19 fu = floor(fu*10^4)/10^4
20 u = fu * 10^Eu
21 w = u + z
22 n = length(string(w))
23 w = floor(w/10^(n-4))*10^(n-4) //To maintain length
   of mantissa = 4
24 disp(w,'w = ')
25 True_w = 10444
26 ew = True_w - w
27 er_w = (True_w - w)/True_w
28 disp(er_w,'er ,w = ',ew,'ew = ',True_w,'True w = ')
```

---

### Scilab code Exa 4.12 Addition of Chain of Numbers

```
1 //Example No. 4_12
2 //Addition of chain Numbers
3 //Pg No. 77
4 clear ; close ; clc ;
5
6 x = 9678 ;
7 y = 678 ;
8 z = 78 ;
9 d = 4 ; //length of mantissa
10 n = max(length( string(y) ) , length(string(z)))
11 fy = y/10^n
12 fz = z/10^n
13 fu = fy + fz
14 Eu = n
15 if fu >= 1 then
16     fu = fu/10
17     Eu = Eu + 1
18 end
19 u = fu * 10^Eu
20 n = max(length( string(x) ) , length(string(u)))
21 fu = u/10^4
22 fx = x/10^4
23 fw = fu + fx
24 Ew = 4
25 if fw >= 1 then
26     fw = fw/10
27     Ew = Ew + 1
28 end
29 //since length of mantissa is only four we need to
   maintain only four places in decimal , so
30 fw = floor(fw*10^4)/10^4
31 w = fw*10^Ew
```

```
32 disp(w, 'w = ')
33 True_w = 10444
34 ew = True_w - w
35 er_w = (True_w - w)/True_w
36 disp(er_w, 'er ,w = ', ew, 'ew = ', True_w, 'True w = ')

---


```

### Scilab code Exa 4.13 Theoritical Problem

```
1 //Example No. 4_13
2 //Pg No. 78
3 disp('Theoritical Problem')
4 disp('For Details go to page no. 78')

---


```

### Scilab code Exa 4.14 Absolute and Relative Errors

```
1 //Example No. 4_14
2 //Absolute & Relative Errors
3 //Pg No. 79
4 clear ; close ; clc ;
5
6 xa = 4.000
7 def(f = f(x) , 'f = sqrt(x) + x')
8 //Assuming x is correct to 4 significant digits
9 ex = 0.5 * 10^(-4 + 1)
10 df_xa = derivative(f,4)
11 ef = ex * df_xa
12 er_f = ef/f(xa)
13 mprintf('\n ex = %.0E \n df(xa) = %.2f \n ef = %.2E
          \n er ,f = %.2E \n', ex,df_xa,ef,er_f)

---


```

### Scilab code Exa 4.15 Error Evaluation

```
1 //Example No. 4_15
2 //Error Evaluation
3 //Pg No. 80
4 clear ; close ; clc ;
5
6 x = 3.00 ;
7 y = 4.00 ;
8 deff('f = f(x,y)', 'f = x^2 + y^2')
9 deff('df_x = df_x(x)', 'df_x = 2*x')
10 deff('df_y = df_y(y)', 'df_y = 2*y')
11 ex = 0.005
12 ey = 0.005
13 ef = df_x(x)*ex + df_y(y)*ey
14 disp(ef, 'ef = ')
```

---

### Scilab code Exa 4.16 Condition and Stability

```
1 //Example No. 4_16
2 //Condition and Stability
3 //Pg No. 82
4 clear ; close ; clc ;
5
6 C1 = 7.00 ;
7 C2 = 3.00 ;
8 m1 = 2.00 ;
9 m2 = 2.01 ;
10 x = (C1 - C2)/(m2 - m1)
11 y = m1*((C1 - C2)/(m2 - m1)) + C1
12 disp(y, 'y = ', x, 'x = ')
13 disp('Changing m2 from 2.01 to 2.005')
14 m2 = 2.005
15 x = (C1 - C2)/(m2 - m1)
16 y = m1*((C1 - C2)/(m2 - m1)) + C1
```

17 **mprintf**(' \n x = %i \n y = %i \n From the above  
results we can see that for small change in m2  
results in almost 100 percent change in the  
values of x and y. Therefore , the problem is  
absolutely ill-conditioned \n ',x,y)

---

#### Scilab code Exa 4.17 Theoritical Problem

```
1 //Example No. 4_17
2 //Pg No. 83
3 disp('Theoritical Problem')
4 disp('For Details go to page no. 83')
```

---

#### Scilab code Exa 4.18 Difference of Square roots

```
1 //Example No. 4_18
2 //Difference of Square roots
3 //Pg No. 84
4 clear ; close ; clc ;
5
6 x = 497.0 ;
7 y = 496.0 ;
8 sqrt_x = sqrt(497)
9 sqrt_y = sqrt(496)
10 nx = length( string( floor( sqrt_x ) ) )
11 ny = length( string( floor( sqrt_y ) ) )
12 sqrt_x = floor(sqrt_x*10^(4-nx))/10^(4-nx)
13 sqrt_y = floor(sqrt_y*10^(4-ny))/10^(4-ny)
14 z1 = sqrt_x - sqrt_y
15 disp(z1,'z = sqrt(x) - sqrt(y)')
16 z2 = ( x -y)/(sqrt_x + sqrt_y)
17 if z2 < 0.1 then
18     z2 = z2*10^4
```

```
19     nz = length(string(floor(z2)))
20     z2 = floor(z2*10^(4-nz))/10^(8-nz)
21 end
22 disp( z2 , 'z = ( x-y )/( sqrt(x) + sqrt(y) )' )
```

---

### Scilab code Exa 4.19 Theoretical Problem

```
1 //Example No. 4_19
2 //Pg No. 84
3 disp('Theoretical Problem')
4 disp('For Details go to page no. 84')
```

---

### Scilab code Exa 4.20 Theoretical Problem

```
1 //Example No. 4_20
2 //Pg No. 85
3 disp('Theoretical Problem')
4 disp('For Details go to page no. 85')
```

---

### Scilab code Exa 4.21 Induced Instability

```
1 //Example 4_21
2 //Pg No. 85
3 clear ; close ; clc ;
4
5 x = -10
6 T_act(1) = 1
7 T_trc(1) = 1
8 e_x_cal = 1
9 for i = 1:100
```

```

10      T_act(i+1) = T_act(i)*x/i
11      T_trc(i+1) = floor(T_act(i+1)*10^5)/10^5
12      TE(i) = abs(T_act(i+1)-T_trc(i+1))
13      e_x_cal = e_x_cal + T_trc(i+1)
14 end
15 e_x_act = exp(-10)
16 disp(e_x_act,'actual e^x = ',e_x_cal,'calculated e^x
    using roundoff = ',sum(TE),'Truncation Error = '
    )
17 disp('Here we can see the difference between
    calculated e^x and actual e^x this is due to
    truncation error (which is greater than final
    value of e^x ), so the roundoff error totally
    dominates the solution')

```

---

# Chapter 6

## Roots of Nonlinear Equations

Scilab code Exa 6.1 Possible initial guess values for roots

```
1 //Example No. 6_01
2 //Possible Initial guess values for roots
3 //Pg No. 126
4
5 clear ; close ; clc ;
6
7 A = [ 2 ; -8 ; 2 ; 12]; // Coefficients of x terms
    in the decreasing order of power
8 n = size(A);
9 x1 = -A(2)/A(1);
10 disp(x1, 'The largest possible root is x1 =')
11 disp(x1, 'No root can be larger than the value')
12
13 x = sqrt((A(2)/A(1))^2 - 2*(A(3)/A(1))^2);
14
15 printf ('\n all real roots lie in the interval (%f,
    %f)\n',x,x)
16 disp('We can use these two points as initial guesses
        for the bracketing methods and one of them for
        open end methods')
```

---

### Scilab code Exa 6.02 Theoretical Problem

```
1 //Example No. 6_02
2 //Pg No. 128
3 disp('Theoretical Problem')
4 disp('For Details go to page no. 128')
```

---

### Scilab code Exa 6.3 Evaluating Polynomial using Horner's rule

```
1 //Example No. 6_03
2 //Evaluating Polynomial using Horner's rule
3 //Pg No.
4 clear ; close ; clc ;
5
6 //Coefficients of x terms in the increasing order of
  power
7 A = [ 6 ; 1 ; -4 ; 1];
8 x = 2
9 [n,c] = size(A) ;
10 p(n) = A(n)
11 disp(p(n), 'p(4) = ')
12 for i = 1:n-1
13     p(n-i) = p(n-i+1)*x + A(n-i)
14     printf ('\n p(%i)= %i\n',n-i,p(n-i))
15 end
16 mprintf ('\n f(%i) = p(1) = %i',x,p(1))
```

---

### Scilab code Exa 6.4 Bisection Method

```

1 //Example No. 6_04
2 //Root of a Equation Using Bisection Method
3 //Pg No. 132
4
5 clear ; close ; clc ;
6
7 //Coefficients in increasing order of power of x
    starting from 0
8 A = [-10 -4 1];
9 disp('First finding the interval that contains a
    root ,this can be done by using Eq 6.10 ')
10 xmax = sqrt((A(2)/A(3))^2 - 2*(A(1)/A(3)))
11 printf('\n Both the roots lie in the interval (%i ,
    %i) \n',xmax,xmax)
12 x = -6:6
13 p = poly(A, 'x' , 'c ')
14 fx = horner(p,x);
15 for i = 1:12
16     if fx(1,i)*fx(1,i+1) < 0 then
17         break ;
18     end
19 end
20 printf('\n The root lies in the interval (%i,%i)\n' ,
    x(1,i),x(1,i+1))
21 x1 = x(1,i);
22 x2 = x(1,i+1);
23 f1 = fx(1,i);
24 f2 = fx(1,i+1);
25 err = abs((x2-x1)/x2) ;
26 while err > 0.0001
27 x0 = (x1 + x2)/2 ;
28 f0 = horner(p,x0);
29 if f0*f1 < 0 then
30     x2 = x0
31     f2 = f0
32 elseif f0*f2 < 0
33     x1 = x0
34     f1 = f0

```

```

35 else
36     break
37 end
38 printf ('\n the root lies in the interval (%f,%f)\n',
39 x1,x2);
40 err = abs((x2-x1)/x2);
41 end
42 printf ('\nthe approximate root is %f\n',x0)

```

---

### Scilab code Exa 6.5 False Position Method

```

1 //Example No. 6_05
2 //False Position Method
3 //Pg No. 139
4 clear ; close ; clc ;
5
6 //Coefficients of polynomial in increasing order of
   power of x
7 A = [-2 -1 1];
8 x1 = 1 ;
9 x2 = 3 ;
10 fx = poly(A, 'x', 'c');
11 for i = 1:15
12     printf ('Iteration No. %i \n',i);
13     fx1 = horner(fx,x1);
14     fx2 = horner(fx,x2);
15     x0 = x1 - fx1*(x2-x1)/(fx2-fx1)
16     printf ('x0 = %f \n',x0);
17     fx0 = horner(fx,x0);
18     if fx1*fx0 < 0 then
19         x2 = x0 ;
20     else
21         x1 = x0 ;
22     end
23 end

```

---

### Scilab code Exa 6.06 Theoretical Problem

```
1 //Example No. 6_06
2 //Pg No. 146
3 disp('Theoretical Problem')
4 disp('For Details go to page no. 146')
```

---

### Scilab code Exa 6.7 Newton Raphson Method

```
1 //Example No. 6_07
2 //Root of the Equation using Newton Raphson Method
3 //Pg No. 147
4 clear ; close ; clc ;
5
6 //Coefficients of polynomial in increasing order of
   power of x
7 A = [ 2 -3 1];
8 fx = poly(A, 'x', 'c');
9 dfx = derivat(fx);
10
11 x(1) = 0 ;
12 for i = 1:10
13     f(i) = horner(fx,x(i));
14     if f(i) ~= 0 then
15         df(i) = horner(dfx,x(i));
16         x(i+1) = x(i) - f(i)/df(i) ;
17         printf('x%i = %f\n',i+1,x(i+1));
18     else
19         printf('Since f(%f) = 0, the root closer to
               the point x = 0 is %f \n',x(i),x(i));
20     break
```

```
21     end  
22 end
```

---

### Scilab code Exa 6.8 Newton Raphson Method

```
1 //Example No. 6_08  
2 //Root of the Equation using Newton Raphson Method  
3 //Pg No. 151  
4 clear ; close ; clc ;  
5 //Coefficients of polynomial in increasing order of  
power of x  
6 A = [ 6 1 -4 1 ];  
7 fx = poly(A, 'x', 'c');  
8 dfx = derivat(fx);  
9  
10 x(1) = 5.0 ;  
11 for i = 1:6  
12     f(i) = horner(fx, x(i));  
13     if f(i) ~= 0 then  
14         df(i) = horner(dfx, x(i));  
15         x(i+1) = x(i) - f(i)/df(i) ;  
16         printf('x%i = %f\n', i+1, x(i+1));  
17     end  
18 end  
19 disp('From the results we can see that number of  
correct digits approximately doubles with each  
iteration')
```

---

### Scilab code Exa 6.9 Secant Method

```
1 //Example No. 6_09  
2 //Root of the equation using SECANT Method  
3 //Pg No. 153
```

```

4 clear ; close ; clc ;
5
6 //Coefficients of polynomial in increasing order of
   power of x
7 A = [ -10 -4 1];
8 x1 = 4 ;
9 x2 = 2 ;
10 fx = poly(A, 'x', 'c')
11 for i = 1:6
12     printf('\n For Iteration No. %i\n',i)
13     fx1 = horner(fx,x1);
14     fx2 = horner(fx,x2);
15     x3 = x2 - fx2*(x2-x1)/(fx2-fx1) ;
16     printf ('\n x1 = %f\n x2 = %f \n fx1 = %f \n fx2
           = %f \n x3 = %f \n ',x1,x2,fx1,fx2,x3) ;
17     x1 = x2;
18     x2 = x3;
19 end
20 disp('This can be still continued further for
accuracy ')

```

---

### Scilab code Exa 6.10 Theoretical Problem

```

1 //Example No. 6_10
2 //Pg No. 155
3 disp('Theoretical Problem')
4 disp('For Details go to page no. 155')

```

---

### Scilab code Exa 6.11 Fixed Point Method

```

1 //Example No. 6_11
2 //Fixed point method
3 //Pg No. 161

```

```

4 clear ; close ; clc ;
5
6 //Coefficients of polynomial in increasing order of
   power of x
7 A = [ -2 1 1 ];
8 B = [ 2 0 -1 ];
9 gx = poly(B, 'x', 'c');
10 x(1) = 0 ; //initial guess x0 = 0
11 for i = 2:10
12     x(i) = horner(gx,x(i-1));
13     printf ('\n x%i = %f\n',i-1,x(i))
14     if (x(i)-x(i-1)) == 0 then
15         printf ('\n%f is root of the equation , since
           x%i - x%i = 0 \n',x(i),i-1,i-2)
16         break
17     end
18 end
19 //Changing initial guess x0 = -1
20 x(1) = -1 ;
21 for i = 2:10
22     x(i) = horner(gx,x(i-1));
23     printf ('\n x%i = %f\n',i-1,x(i))
24     if (x(i)-x(i-1)) == 0 then
25         printf ('\n %f is root of the equation , since
           x%i - x%i = 0 ',x(i),i-1,i-2)
26         break
27     end
28 end

```

---

### Scilab code Exa 6.12 Fixed Point Method

```

1 //Example No. 6_12
2 //Fixed point method
3 //Pg No. 162
4 clear ; close ; clc ;

```

```

5
6 A = [ -5 0 1 ];
7 funcprot(0);
8 def(x = g(x), 'x = 5/x');
9 x(1) = 1 ;
10 printf ('\n x0 = %f \n',x(1));
11 for i = 2:5
12     x(i) = feval(x(i-1),g);
13     printf (' x%i = %f \n',i-1,x(i))
14 end
15 //Defining g(x) in different way
16 def(x = g(x), 'x = x^2 + x - 5');
17 x(1) = 0;
18 printf ('\n x0 = %f \n',x(1));
19 for i = 2:5
20     x(i) = feval(x(i-1),g);
21     printf (' x%i = %f \n',i-1,x(i))
22 end
23 //Third form of g(x)
24 def(x = g(x), ' x = (x + 5/x)/2 ');
25 x(1) = 1;
26 printf ('\n x0 = %f \n',x(1));
27 for i = 2:7
28     x(i) = feval(x(i-1),g);
29     printf (' x%i = %f \n',i-1,x(i))
30 end

```

---

### Scilab code Exa 6.13 Fixed Point Method for non linear equations

```

1 //Example No. 6_13
2 //Solving System of non-linear equations using FIXED
   POINT METHOD
3 //Pg No. 169
4 clear ; close ; clc ;
5

```

```

6 printf(' x^2 - y^2 = 3 \n x^2 + x*y \n');
7 deff('x = f(x,y)', 'x = y + 3/(x+y)' );
8 deff('y = g(x)', 'y = (6-x^2)/x' );
9 x(1) = 1 ;
10 y(1) = 1 ;
11 printf ('\n x0 = %f \n y0 = %f \n',x(1),y(1));
12 for i = 2:4
13     x(i) = feval(x(i-1),y(i-1),f);
14     y(i) = feval(x(i-1),g);
15     printf ('\n x%i = %f \n y%i = %f \n',i-1,x(i),i
16     -1,y(i));
16 end

```

---

### Scilab code Exa 6.14 Newton Raphson Method for Non linear equations

```

1 //Example No. 6_14
2 //Solving System of Non-linear equations using
   Newton Raphson Method
3 //Pg No. 172
4 clear ; close ; clc ;
5
6 printf('x^2 + x*y = 6 \n x^2 - y^2 = 3 \n');
7 deff('f = F(x,y)', 'f = x^2 + x*y - 6' );
8 deff('g = G(x,y)', 'g = x^2 - y^2 -3' );
9 deff('f1 = dFx(x,y)', 'f1 = 2*x + y' );
10 deff('f2 = dFy(x,y)', 'f2 = y' );
11 deff('g1 = dGx(x,y)', 'g1 = 2*x' );
12 deff('g2 = dGy(x,y)', 'g2 = -2*y' );
13 x(1) = 1 ;
14 y(1) = 1 ;
15
16 for i = 2:3
17     Fval = feval(x(i-1),y(i-1),F);
18     Gval = feval(x(i-1),y(i-1),G);
19     f1 = feval(x(i-1),y(i-1),dFx);

```

```

20     f2 = feval(x(i-1),y(i-1),dFy);
21     g1 = feval(x(i-1),y(i-1),dGx);
22     g2 = feval(x(i-1),y(i-1),dGy);
23     D = f1*g2 - f2*g1 ;
24
25     x(i) = x(i-1) - (Fval*g2 - Gval*f2)/D ;
26     y(i) = y(i-1) - (Gval*f1 - Fval*g1)/D ;
27     printf ('\n x%i = %f \n y%i = %f \n',i-1,x(i),i
28                           -1,y(i))
29 end

```

---

### Scilab code Exa 6.15 Synthetic Division

```

1 //Example No. 6_15
2 //Synthetic Division
3 //Pg No. 176
4 clear ; close ; clc ;
5
6 a = [-9 15 -7 1];
7 b(4) = 0 ;
8 for i = 3:-1:1
9     b(i) = a(i+1) + b(i+1)*3
10    printf ('b%i = %f\n',i,b(i))
11 end
12 disp(poly(b,'x','c')) , 'Thus the polynomial is ')

```

---

### Scilab code Exa 6.16 Bairstow Method for Factor of polynomial

```

1 //Example No. 6_16
2 //Quadratic factor of a polynomial using Bairstow's
   Method
3 //Pg No. 187

```

```

4 clear ; close ; clc ;
5
6 a = [ 10 1 0 1];
7 n = length(a);
8 u = 1.8 ;
9 v = -1 ;
10
11 b(n) = a(n);
12 b(n-1) = a(n-1) + u*b(n);
13 c(n) = 0 ;
14 c(n-1) = b(n);
15
16 for i = n-2:-1:1
17     b(i) = a(i) + u*b(i+1) + v*b(i+2) ;
18     c(i) = b(i+1) + u*c(i+1) + v*c(i+2) ;
19 end
20 for i = n:-1:1
21     printf('b%di = %f \n',i-1,b(i))
22 end
23 for i = n:-1:1
24     printf('c%di = %f \n',i-1,c(i))
25 end
26
27 D = c(2)*c(2) - c(1)*c(3) ;
28 du = -1*(b(2)*c(2) - c(1)*c(3))/D ;
29 dv = -1*(b(1)*c(2) - b(2)*c(1))/D ;
30 u = u + du ;
31 v = v + dv ;
32 printf('\n D = %f \n du = %f \n dv = %f \n u = %f\n
v = %f \n',D,du,dv,u,v)

```

---

### Scilab code Exa 6.17 Mullers Method for Leonards equation

```

1 //Example No. 6_17
2 //Solving Leonard's equation using MULLER'S Method

```

```

3 //Pg No. 197
4 clear ; close ; clc ;
5
6 def( 'y = f(x)', 'y = x^3 + 2*x^2 + 10*x - 20' ) ;
7 x1 = 0 ;
8 x2 = 1 ;
9 x3 = 2 ;
10 for i = 1:10
11     f1 = feval(x1,f) ;
12     f2 = feval(x2,f) ;
13     f3 = feval(x3,f) ;
14     h1 = x1-x3 ;
15     h2 = x2-x3 ;
16     d1 = f1 - f3 ;
17     d2 = f2 - f3 ;
18     D = h1*h2*(h1-h2);
19     a0 = f3 ;
20     a1 = (d2*h1^2 - d1*h2^2)/D ;
21     a2 = (d1*h2 - d2*h1)/D ;
22     if abs(-2*a0/( a1 + sqrt( a1^2 - 4*a0*a2 ) )) <
23         abs( -2*a0/( a1 - sqrt( a1^2 - 4*a0*a2 ) ) )
24         h4 = -2*a0/(a1 + sqrt(a1^2 - 4*a0*a2));
25     else
26         h4 = -2*a0/(a1 - sqrt(a1^2 - 4*a0*a2))
27     end
28     x4 = x3 + h4 ;
29     printf( '\n x1 = %f\n x2 = %f\n x3 = %f\n f1 = %f
30             \n f2 = %f\n f3 = %f\n h1 = %f\n h2 = %f\n d1
31             = %f\n d2 = %f\n a0 = %f\n a1 = %f\n a2 = %f
32             \n h4 = %f\n x4 = %f\n ',x1,x2,x3,f1,f2,f3,h1
33             ,h2,d1,d2,a0,a1,a2,h4,x4) ;
34     relerr = abs((x4-x3)/x4);
35     if relerr <= 0.00001
36         printf('root of the polynomial is x4 = %f',
37                 x4);
38         break
39     end
40     x1 = x2 ;

```

```
35      x2 = x3 ;
36      x3 = x4 ;
37
38
39 end
```

---

# Chapter 7

## Direct Solutions of Linear Equations

Scilab code Exa 7.1 Elimination Process

```
1 //Example No. 7_01
2 //Elimination Process
3 //Pg No. 211
4
5 clear ; close ; clc ;
6
7 A = [3 2 1 10; 2 3 2 14 ; 1 2 3 14];
8 A(2,:) = A(2,:)-A(1,:)*A(2,1)/A(1,1)
9 A(3,:) = A(3,:)-A(1,:)*A(3,1)/A(1,1)
10 disp(A)
11
12 A(3,:) = A(3,:)-A(2,:)*A(3,2)/A(2,2)
13 disp(A)
14
15 z = A(3,4)/A(3,3)
16 y = (A(2,4) - A(2,3)*z)/A(2,2)
17 x = (A(1,4) - A(1,2)*y - A(1,3)*z)/A(1,1)
18 disp(x, 'x = ', y, 'y = ', z, 'z = ')
```

---

### Scilab code Exa 7.2 Basic Gauss Elimination

```
1 //Example No. 7_02
2 //Basic Gauss Elimination
3 //Pg No. 214
4 clear ; close ; clc ;
5
6 A = [ 3   6   1   ;   2   4   3   ;   1   3   2 ];
7 B = [16 13 9];
8 [ar1,ac1] = size(A);
9 Aug = [3 6 1 16 ; 2 4 3 13 ; 1 3 2 9]
10 for i = 2 : ar1
11     Aug(i,:) = Aug(i,:)- (Aug(i,1)/Aug(1,1))*Aug
12         (1,:) ;
13 end
14 disp(Aug)
15 disp('since Aug(2,2) = 0 elimination is not possible
16 , so reordering the matrix')
17 Aug = Aug( [ 1 3 2],:);
18 disp(Aug)
19 disp('Elimination is complete and by back
20 substitution the solution is\n')
21 disp('x3 = 1, x2 = 2 , x1 = 1 ')
```

---

### Scilab code Exa 7.3 Gauss Elimination using Partial Pivoting

```
1 //Example No. 7_03
2 // Gauss Elimination using partial pivoting
3 // Pg No. 220
4 clear ; close ; clc ;
5
6 A = [ 2   2   1   ;   4   2   3   ;   1   -1   1];
```

```

7 B = [ 6   ;   4   ;   0   ];
8 [ ar , ac ] = size(A);
9 Aug = [ 2   2   1   6   ;   4   2   3   4   ;   1   -1   1
          0   ];
10
11 for i = 1 : ar-1
12     [ p , m ] = max(abs(Aug(i:ar,i)))
13     Aug(i:ar,:) = Aug([i+m-1 i+1:i+m-2 i i+m:ar
                           ],:);
14     disp(Aug)
15     for k = i+1 : ar
16         Aug(k,i:ar+1) = Aug(k,i:ar+1) - (Aug(k,i)/
17                                         Aug(i,i))*Aug(i,i:ar+1);
18     end
19     disp(Aug)
20 end
21 //Back Substitution
22 X(ar,1) = Aug(ar,ar+1)/Aug(ar,ar)
23 for i = ar-1 : -1 : 1
24     X(i,1) = Aug(i,ar+1);
25     for j = ar : -1 : i+1
26         X(i,1) = X(i,1) - X(j,1)*Aug(i,j);
27     end
28     X(i,1) = X(i,1)/Aug(i,i);
29 end
30
31 printf('\n The solution can be obtained by back
           substitution \n x1 = %i \n x2 = %i \n x3 = %i \n',
           ,X(1,1),X(2,1),X(3,1))

```

---

### Scilab code Exa 7.4 Gauss Jordan Elimination

```

1 //Example No. 7_04
2 //Gauss Jordan Elimination

```

```

3 //Pg No. 228
4 clear ; close ; clc ;
5
6 A = [ 2 4 -6 ; 1 3 1 ; 2 -4 -2 ] ;
7 B = [ -8 ; 10 ; -12 ] ;
8 [ar , ac] = size(A) ;
9 Aug = [ 2 4 -6 -8 ; 1 3 1 10 ; 2 -4 -2
          -12 ] ;
10 disp(Aug)
11
12 for i = 1 : ar
13     Aug(i,i:ar+1) = Aug(i,i:ar+1)/Aug(i,i) ;
14     disp(Aug)
15     for k = 1 : ar
16         if k ~= i then
17             Aug(k,i:ar+1) = Aug(k,i:ar+1) - Aug(k,i)
                           *Aug(i,i:ar+1);
18         end
19     end
20     disp(Aug)
21 end

```

---

### Scilab code Exa 7.5 DoLittle LU Decomposition

```

1 //Example No. 7_05
2 //DoLittle LU Decomposition
3 //Pg No. 234
4
5 clear ; close ; clc ;
6
7 A = [ 3 2 1 ; 2 3 2 ; 1 2 3 ] ;
8 B = [ 10 ; 14 ; 14 ] ;
9 [n , n] = size(A) ;
10
11 for i = 2:n

```

```

12     U(1,:) = A(1,:);
13     L(i,i) = 1 ;
14     if i ~= 1 then
15         L(i,1) = A(i,1)/U(1,1);
16     end
17 end
18
19 for j = 2:n
20     for i = 2:n
21
22         if i <= j then
23             U(i,j) = A(i,j);
24             for k = 1:i-1
25                 U(i,j) = U(i,j) - L(i,k)*U(k,j);
26             end
27             printf ('\nU(%i,%i) = %f \n',i,j,U(i,j))
28
29         else
30             L(i,j) = A(i,j)
31             for k = 1:j-1
32                 L(i,j) = L(i,j) - L(i,k)*U(k,j);
33             end
34             L(i,j) = L(i,j)/U(j,j) ;
35             printf ('\n L(%i,%i) = %f \n',i,j,L(i,j))
36         end
37     end
38 end
39 disp(U, 'U = ')
40 disp(L, 'L = ')
41
42 //Forward Substitution
43 for i = 1:n
44     z(i,1) = B(i,1);
45     for j = 1:i-1
46         z(i,1) = z(i,1) - L(i,j)*z(j,1);
47     end
48     printf ('\n z(%i) = %f \n',i,z(i,1))
49 end

```

```

50
51 //Back Substitution
52 for i = n : -1 : 1
53     x(i,1) = z(i,1);
54     for j = n : -1 : i+1
55         x(i,1) = x(i,1) - U(i,j)*x(j,1);
56     end
57     x(i,1) = x(i,1)/U(i,i);
58     printf ('\n x(%i) = %f \n',i,x(i,1))
59 end

```

---

### Scilab code Exa 7.6 Choleskys Factorisation

```

1 //Example No. 7_06
2 //Cholesky 's Factorisation
3 //Pg No. 242
4
5 clear ; close ; clc ;
6
7 A = [ 1 2 3 ; 2 8 22 ; 3 22 82 ];
8 [n,n] = size(A);
9
10 for i = 1:n
11     for j = 1:n
12         if i == j then
13             U(i,i) = A(i,i)
14             for k = 1:i-1
15                 U(i,i) = U(i,i)-U(k,i)^2 ;
16             end
17             U(i,i) = sqrt(U(i,i));
18         elseif i < j
19             U(i,j) = A(i,j)
20             for k = 1:i-1
21                 U(i,j) = U(i,j) - U(k,i)*U(k,j);
22             end

```

```
23           U(i,j) = U(i,j)/U(i,i)
24       end
25   end
26 end
27 disp(U)
```

---

### Scilab code Exa 7.7 Ill Conditioned Systems

```
1 //Example No. 7_07
2 //Ill-Conditioned Systems
3 //Pg No. 245
4
5 clear ; close ; clc ;
6
7 A = [ 2   1 ; 2.001  1];
8 B = [ 25 ; 25.01 ];
9 x(1) = (25 - 25.01)/(2 - 2.001);
10 x(2) = (25.01*2 - 25*2.001)/(2*1 - 2.001*1);
11 printf ('\n x(1) = %f \n x(2) = %f \n',x(1),x(2))
12 x(1) = (25 - 25.01)/(2 - 2.0005);
13 x(2) = (25.01*2 - 25*2.0005)/(2*1 - 2.0005*1);
14 printf ('\n x(1) = %f \n x(2) = %f \n',x(1),x(2))
15 r = A*x-B
16 disp(x)
17 disp(r)
```

---

# Chapter 8

## Iterative Solution of Linear Equations

Scilab code Exa 8.1 Gauss Jacobi Iteration Method

```
1 //Example No. 8_01
2 //Gauss Jacobi
3 //Page No. 254
4 clear ; close ; clc ;
5
6 A = [ 2   1   1 ; 3   5   2 ; 2   1   4];
7 B = [ 5   ; 15  ; 8];
8 x1old = 0 ,x2old = 0 , x3old = 0 // intial assumption
of x1 ,x2 & x3
9
10 disp('x1 = (5 - x2 - x3)/2 ')
11 disp('x2 = (15 - 3x1 - 2x3)/5 ')
12 disp('x3 = (8 - 2x1 - x2)/4 ')
13
14 for i = 1:4
15     printf('\n Iteration Number : %d\n',i)
16
17     x1 = (5 - x2old - x3old)/2 ;
18     x2 = (15 - 3*x1old - 2*x3old)/5 ;
```

```

19     x3 = (8 - 2*x1old - x2old)/4 ;
20
21     printf (' \n x1 = %f\n x2 = %f\n x3 = %f\n ',x1,x2
22           ,x3)
23
24     x1old = x1;
25     x2old = x2;
26     x3old = x3;
27 end

```

---

### Scilab code Exa 8.2 Gauss Seidel Iterative Method

```

1 //Example No. 8_02
2 //Gauss Seidel
3 //Page No.261
4 clear ; close ; clc ;
5
6 A = [ 2   1   1 ; 3   5   2 ; 2   1   4];
7 B = [ 5   ; 15  ; 8];
8 x1old = 0 ,x2old = 0 , x3old = 0 // intial assumption
9
10 disp( '(x1 = 5 - x2 - x3)/2 ')
11 disp( '(x2 = 15 - 3x1 - 2x3)/5 ')
12 disp( '(x3 = 8 - 2x1 - x2)/4 ')
13
14 for i = 1:2
15
16     printf ('\n Iteration Number : %d',i)
17
18     x1 = (5 - x2old - x3old)/2 ;
19     x1old = x1;
20     x2 = (15 - 3*x1old - 2*x3old)/5 ;
21     x2old = x2;
22     x3 = (8 - 2*x1old - x2old)/4 ;

```

```

23     x3old = x3;
24
25     printf( '\n x1 = %f\n x2 = %f\n x3 = %f\n' ,x1 ,
26             x2 ,x3)
27 end

```

---

### Scilab code Exa 8.3 Gauss Seidel Iterative Method

```

1 //Example No. 8_03
2 //Gauss Seidel
3 //page no. 269
4 clear ; close ; clc ;
5
6 A = [ 3 1 ; 1 -3]
7 B = [ 5 ; 5 ]
8
9 disp('Using a matrix to display the results after
       each iteration , first row represents initial
       assumption')
10 X(1,1) = 0 , X(1,2) = 0 ;// initial assumption
11
12 maxit = 1000; //Maximum number of iterations
13 err = 0.0003 ;
14
15 disp('x1 = (5-x2)/3');
16 disp('x2 = (x1 - 5)/3');
17
18 for i = 2:maxit
19
20     X(i,1) = (5 - X(i-1,2))/3 ;
21     X(i,2) = (X(i,1) - 5)/3 ;
22
23 //Error Calculations
24 err1 =abs((X(i,1) - X(i-1,1))/X(i,1))

```

```

25     err2 =abs((X(i,2)- X(i-1,2))/X(i,2))
26
27 //Terminating Condition
28 if err >= err1 & err >= err2 then
29     printf('The system converges to the solution
30         ( %f , %f ) in %d iterations\n',X(i,1),X
31             (i,2),i-1 )
32     break
33 end
34 //calcution of true error i.e. difference between
35     final result and results from each iteration
35 trueerr1 = abs(X(:,1) - X(i,1)*ones(i,1)) ;
36 trueerr2 = abs(X(:,2) - X(i,2)*ones(i,1)) ;
37
38 //displaying final results
39 D = [ X trueerr1 trueerr2] ;
40 disp(D)

```

---

### Scilab code Exa 8.4 Gauss Seidel Iterative Method

```

1 //Example No. 8_04
2 //Gauss Seidel
3 //Page No.261
4 clear ; close ; clc ;
5
6 A = [ 1 -3 ; 3 1 ];
7 B = [ 5 ; 5 ];
8 x1old = 0 ,x2old = 0 //intial assumption
9
10 disp( 'x1 = 5 + 3*x2 ')
11 disp( 'x2 = 5 - 3*x1 ')
12
13 for i = 1:3

```

```
14
15      x1 = 5 + 3*x2old ;
16      x1old = x1;
17      x2 = 5 - 3*x1old ;
18      x2old = x2;
19
20      printf ('\n Iteration : %i    x1 = %i and x2 = %i\
n',i,x1,x2)
21
22 end
23 disp('It is clear that the process do not converge
towards the solution , rather it diverges .')
```

---

# Chapter 9

## Curve Fitting Interpolation

Scilab code Exa 9.1 Polynomial Forms

```
1 //Example No. 9_01
2 //Pg No.277
3 clear ; close ; clc ;
4
5 printf('solving linear equations \n a0 + 100a1 = 3/7
         \n a0 + 101a1 = -4/7 \n we get ,\n');
6 C = [ 1 100 ; 1 101]
7 p = [ 3/7 ; -4/7]
8 a = C\p
9 printf ('\n a0 = %f \n a1 = %f \n ',a(1),a(2));
10 x = poly(0,'x') ;
11 px = a(1) + a(2)*x
12 p100 = horner(px,100)
13 p101 = horner(px,101)
14 printf ('\n p(100) = %f \n p(101) = %f\n',p100,p101)
```

---

Scilab code Exa 9.2 Shifted Power form

```

1 //Example No. 9_02
2 //Page No. 278
3 clear ; close ; clc ;
4
5 C = [ 1 100-100 ; 1 101-100]
6 p = [ 3/7 ; -4/7]
7 a = C\p
8 printf ('\n a0 = %f \n a1 = %f \n',a(1),a(2));
9 x = poly(0,'x') ;
10 px = a(1) + a(2)*(x - 100)
11 p100 = horner(px,100)
12 p101 = horner(px,101)
13 printf ('\n p(100) = %f \n p(101) = %f\n',p100,p101)

```

---

### Scilab code Exa 9.3 Linear Interpolation

```

1 //Example No. 9_03
2 //Page No. 280
3 clear ; close ; clc ;
4
5 x = 1:5
6 f = [1 1.4142 1.7321 2 2.2361]
7 n = 2.5
8 for i = 1:5
9     if n <= x(i) then
10         break ;
11     end
12 end
13 printf ('%f lies between points %i and %i',n,x(i-1),x(i))
14 f2_5 = f(i-1) + ( n - x(i-1) )*( f(i) - f(i-1) )/( x(i) - x(i-1) )
15 err1 = 1.5811 - f2_5
16 disp(f2_5,'f(2.5) = ')
17 disp(err1,'error1 = ')

```

```

18 disp('The correct answer is 1.5811.The difference
      between results is due to use of a linear model
      to a nonlinear use')
19 disp('repeating the procedure using x1 = 2 and x2 =
      4')
20 x1 = 2
21 x2 = 4
22 f2_5 = f(x1) + ( 2.5 - x1 )*( f(x2) - f(x1) )/( x2 -
      x1 )
23 err2 = 1.5811 - f2_5
24 disp(err2, 'error2 = ')
25 disp(f2_5, 'f(2.5) = ')
26 disp('NOTE— The increase in error due to the
      increase in the interval between the
      interpolating data')

```

---

### Scilab code Exa 9.4 Lagrange Interpolation

```

1 //Example No. 9_04
2 //Lagrange Interpolation – Second order
3 //Pg No. 282
4 clear ; close ; clc ;
5
6 X = [ 1 2 3 4 5]
7 Fx = [ 1 1.4142 1.7321 2 2.2361];
8 X = X(2:4)
9 Fx = Fx(2:4)
10 x0 = 2.5
11 x = poly(0, 'x')
12 p = 0
13 for i = 1:3
14     L(i) = 1
15     for j = 1:3
16         if j == i then
17             continue ;

```

```

18     else
19         L(i) = L(i)*(x - X(j))/(X(i) - X(j))
20         ;
21     end
22     p = p + Fx(i)*L(i)
23 end
24 L0 = horner(L(1), 2.5);
25 L1 = horner(L(2), 2.5);
26 L2 = horner(L(3), 2.5);
27 p2_5 = horner(p, 2.5);
28 printf('For x = 2.5 we have,\n L0(2.5) = %f\n L1
    (2.5) = %f\n L2(2.5) = %f\n p(2.5) = %f\n', L0,
    L1, L2, p2_5)
29
30 err = sqrt(2.5) - p2_5;
31 printf('The error is %f', err);

```

---

### Scilab code Exa 9.5 Lagrange Interpolation

```

1 //Example No. 9_05
2 //Lagrange Interpolation
3 //Pg No. 283
4 clear ; close ; clc ;
5
6 i = [ 0 1 2 3 ]
7 X = [ 0 1 2 3 ]
8 Fx = [ 0 1.7183 6.3891 19.0855 ]
9 x = poly(0, 'x');
10 n = 3 //order of lagrange polynomial
11 p = 0
12 for i = 1:n+1
13     L(i) = 1
14     for j = 1:n+1
15         if j == i then

```

```

16         continue ;
17     else
18         L(i) = L(i)*( x - X(j) )/( X(i) - X(j) )
19             ;
20     end
21     p = p + Fx(i)*L(i)
22 end
23 disp("The Lagrange basis polynomials are")
24 for i = 1:4
25     disp(string(L(i)))
26 end
27 disp("The interpolation polynomial is ")
28 disp(string(p))
29 disp('The interpolation value at x = 1.5 is ')
30 p1_5 = horner(p,1.5);
31 e1_5 = p1_5 + 1 ;
32 disp(e1_5, 'e ^1.5 = ', p1_5);

```

---

### Scilab code Exa 9.6 Newton Interpolation

```

1 //Example No. 9_06
2 //Newton Interpolation – Second order
3 //Pg No. 288
4 clear ; close ; clc ;
5
6 i = [ 0 1 2 3]
7 X = 1:4
8 Fx = [ 0 0.3010 0.4771 0.6021]
9 X = 1:3
10 Fx = Fx(1:3)
11 x = poly(0, 'x');
12 A = Fx'
13 for i = 2:3
14     for j = 1:4-i

```

```

15      A(j,i) = ( A(j+1,i-1)-A(j,i-1) )/(X(j+i-1)-X
16          (j)) ;
16      end
17 end
18 printf('The coefficients of the polynomial are,\n a0
19 = %.4G \n a1 = %.4G \n a2 = %.4G \n',A(1,1),A
20 (1,2),A(1,3))
21 p = A(1,1);
22 for i = 2:3
23     p = p +A(1,i)* prod(x*ones(1,i-1) - X(1:i-1));
24 end
25 disp(string(p))
24 p2_5 = horner(p,2.5)
25 printf('p(2.5) = %.4G \n',p2_5)

```

---

### Scilab code Exa 9.7 Newton Divided Difference Interpolation

```

1 //Example No. 9_07
2 //Newton Divided Difference Interpolation
3 //Pg No. 291
4 clear ; close ; clc ;
5
6 i = 0:4
7 X = 1:5
8 Fx = [ 0 7 26 63 124];
9 x = poly(0,'x');
10 A = [ i' X' Fx' ]
11 for i = 4:7
12     for j = 1:8-i
13         A(j,i) = ( A(j+1,i-1)-A(j,i-1) )/(X(j+i-3)-X
14             (j)) ;
14     end
15 end
16 disp(A)
17 p = A(1,3);

```

```

18 p1_5(1) = p ;
19 for i = 4:7
20     p = p +A(1,i)* prod(x*ones(1,i-3) - X(1:i-3));
21     p1_5(i-2) = horner(p,1.5);
22 end
23 printf('p0(1.5) = %f \n p1(1.5) = %f \n p2(1.5) = %f
    \n p3(1.5) = %f \n p4(1.5) = %f \n ',p1_5(1),p1_5
(2),p1_5(3),p1_5(4),p1_5(5));
24 disp(p1_5(5),'The function value at x = 1.5 is ')

```

---

### Scilab code Exa 9.8 Newton Gregory Forward Difference Formula

```

1 //Example No. 9_08
2 //Newton-Gregory forward difference formula
3 //Pg No. 297
4 clear ; close ; clc ;
5
6 X = [ 10 20 30 40 50]
7 Fx = [ 0.1736 0.3420 0.5000 0.6428 0.7660]
8 x = poly(0,'x');
9 A = [X' Fx'];
10 for i = 3:6
11     A(1:7-i,i) = diff(A(1:8-i,i-1))
12 end
13 disp(A)
14 x0 = X(1);
15 h = X(2) - X(1) ;
16 x1 = 25
17 s = (x1 - x0)/h ;
18 p(1) = Fx(1);
19 for j = 1:4
20     p(j+1) = p(j) + prod(s*ones(1,j)-[0:j-1])*A(1,j
        +2)/factorial(j)
21 end
22 printf('p1(s) = %.4G \n p2(s) = %.4G \n p3(s) = %.4G

```

```
\n p4(s) = %.4G \n ,p(2),p(3),p(4),p(5))  
23 printf(' Thus sin(%d) = %.4G \n ',x1,p(5))
```

---

### Scilab code Exa 9.9 Newton Backward Difference Formula

```
1 //Example No. 9_09  
2 //Newton Backward difference formula  
3 //Pg No. 299  
4 clear ;close ;clc ;  
5  
6 X = [ 10 20 30 40 50]  
7 Fx = [ 0.1736 0.3420 0.5000 0.6428 0.7660]  
8 x = poly(0,'x');  
9 A = [X' Fx'];  
10 for i = 3:6  
11     A(i-1:5,i) = diff(A(i-2:5,i-1))  
12 end  
13 disp(A);  
14 xn = X(5);  
15 h = 10 ;  
16 xuk = 25;  
17 s = (xuk - xn)/h ;  
18 disp(s, 's = ');  
19 p(1) = Fx(5)  
20 for j = 1:4  
21     p(j+1) = p(j) + prod(s*ones(1,j)-[0:j-1])*A(5,j  
22         +2)/factorial(j)  
23 end  
23 printf('\n\n p1(s) = %.4G \n p2(s) = %.4G \n p3(s) =  
24         %.4G \n p4(s) = %.4G \n ,p(2),p(3),p(4),p(5))  
24 printf(' Thus sin(%d) = %.4G \n ',xuk,p(5))
```

---

### Scilab code Exa 9.10 Splines

```

1 //Example No. 9_10
2 //Splines
3 //Pg No. 301
4 clear ; close ; clc ;
5
6 x = poly(0, 'x');
7 function isitspline(f1,f2,f3,x0,x1,x2,x3)
8     n1 = degree(f1),n2 = degree(f2),n3 = degree(f3)
9     n = max(n1,n2,n3)
10    f1_x1 = horner(f1,x1)
11    f2_x1 = horner(f2,x1)
12    f2_x2 = horner(f2,x2)
13    f3_x2 = horner(f3,x2)
14    if n ==1 & f1_x1 == f2_x1 & f2_x2 == f3_x2 then
15        printf('The piecewise polynomials are
               continuous and f(x) is a linear spline')
16    elseif f1_x1 == f2_x1 & f2_x2 == f3_x2
17        for i = 1:n-1
18            df1 = derivat(f1)
19            df2 = derivat(f2)
20            df3 = derivat(f3)
21            df1_x1 = horner(df1,x1)
22            df2_x1 = horner(df2,x1)
23            df2_x2 = horner(df2,x2)
24            df3_x2 = horner(df3,x2)
25            f1 = df1, f2 = df2, f3 = df3
26            if df1_x1 ~= df2_x1 | df2_x2 ~= df3_x2
27                then
28                    printf('The %ith derivative of
                           polynomial is not continuours',i)
29                    break
30                end
31            if i == n-1 & df1_x1 == df2_x1 & df2_x2 ==
32                df3_x2 then
33                printf('The polynomial is continuous and
                     its derivatives from 1 to %i are
                     continuous , f(x) is a %ith degree

```

```

                polynomial ',i,i+1)
33         end
34     else
35         printf('The polynomial is not continuous
            ')
36     end
37
38 endfunction
39 n = 4 , x0 = -1 , x1 = 0 , x2 = 1 , x3 = 2
40 f1 = x+1 ;
41 f2 = 2*x + 1 ;
42 f3 = 4 - x ;
43 disp('case 1')
44 isitspline(f1,f2,f3,x0,x1,x2,x3)
45 n = 4, x0 = 0 , x1= 1 , x2 = 2 , x3 = 3
46 f1 = x^2 + 1 ;
47 f2 = 2*x^2 ;
48 f3 = 5*x - 2 ;
49 disp('case 2')
50 isitspline(f1,f2,f3,x0,x1,x2,x3)
51 n = 4, x0 = 0 , x1 = 1, x2 = 2, x3 = 3
52 f1 = x,
53 f2 = x^2 - x + 1,
54 f3 = 3*x - 3
55 disp('case 3')
56 isitspline(f1,f2,f3,x0,x1,x2,x3)

```

---

### Scilab code Exa 9.11 Cubic Spline Interpolation

```

1 //Example No. 9_11
2 //Cubic Spline Interpolation
3 //Pg No. 306
4 clear ; close ; clc ;
5
6 X = [ 4 9 16]

```

```

7 Fx = [ 2 3 4]
8 n = length(X)
9 h = diff(X)
10 disp(h)
11 x = poly(0, 'x');
12 A(1) = 0;
13 A(n) = 0;
14
15 // Since we do not know only a1 = A(2) we just have
   one equation which can be solved directly without
   solving tridiagonal matrix
16 A(2) = 6*( ( Fx(3) - Fx(2) )/h(2) - ( Fx(2) - Fx(1)
   )/h(1) )/( 2*( h(1) + h(2) ) );
17 disp(A(2), 'a1 = ');
18 xuk = 7;
19 for i = 1:n-1
20     if xuk <= X(i+1) then
21         break;
22     end
23 end
24 u = x*ones(1,2) - X(i:i+1)
25 s = ( A(2)*( u(i)^3 - ( h(i)^2 )*u(i) )/6*h(i) ) +
   ( Fx(i+1)*u(i) - Fx(i)*u(i+1) )/h(i);
26 disp(s, 's(x) = ');
27 s_7 = horner(s, xuk);
28 disp(s_7, 's(7)')

```

---

### Scilab code Exa 9.12 Cubic Spline Interpolation

```

1 //Example No. 9_12
2 //Cubic Spline Interpolation
3 //Pg No. 313
4 clear ; close ;clc ;
5
6 X = 1:4 ;

```

```

7 Fx = [ 0.5 0.3333 0.25 0.2]
8 n = length(X)
9 h = diff(X)
10 disp(h)
11 x = poly(0, 'x');
12 A(1) = 0;
13 A(n) = 0;
14 //Forming Tridiagonal Matrix
15 //take make diagonal below main diagonal be 1 , main
      diagonal is 2 and diagonal above main diagonal
      is 3
16 diag1 = h(2:n-2);
17 diag2 = 2*(h(1:n-2)+h(2:n-1));
18 diag3 = h(2:n-2);
19 TridiagMat = diag(diag1,-1)+diag(diag2)+diag(diag3
      ,1)
20 disp(TridiagMat);
21 D = diff(Fx);
22 D = 6*diff(D./h);
23 disp(D)
24 A(2:n-1) = TridiagMat\D'
25 disp(A)
26 xuk = 2.5;
27 for i = 1:n-1
28     if xuk <= X(i+1) then
29         break;
30     end
31 end
32 u = x*ones(1,2) - X(i:i+1)
33 s = ( A(i)*( h(i+1)^2*u(2) - u(2)^2 )/( 6*h(i+1) )
      ) + ( A(i+1)*( u(1)^3 - ( h(i)^2 )*u(1) )/6*h(i)
      ) + ( Fx(i+1)*u(1) - Fx(i)*u(2) )/h(i);
34 disp(s, 's(x) = ');
35 s2_5 = horner(s,xuk);
36 disp(s2_5, 's(2.5)')

```

---

# Chapter 10

## Curve Fitting Regression

Scilab code Exa 10.1 Fitting a Straight line

```
1 //Example No. 10_01
2 //Fitting a Straight Line
3 //Pg No. 326
4 clear ;close ;clc ;
5
6 x = poly(0, 'x')
7 X = 1:5
8 Y = [ 3 4 5 6 8 ];
9 n = length(X);
10 b = ( n*sum(X.*Y) - sum(X)*sum(Y) )/( n*sum(X.*X) -
    (sum(X))^2 )
11 a = sum(Y)/n - b*sum(X)/n
12 disp(b, 'b = ')
13 disp(a, 'a = ')
14 y = a + b*x
```

---

Scilab code Exa 10.2 Fitting a Power Function Model to given data

```

1 //Example No. 10_02
2 //Fitting a Power-Function model to given data
3 //Pg No. 331
4 clear ;close ;clc ;
5
6 x = poly(0, 'x');
7 X = 1:5
8 Y = [ 0.5 2 4.5 8 12.5 ]
9 Xnew = log(X)
10 Ynew = log(Y)
11 n = length(Xnew)
12 b = ( n*sum(Xnew.*Ynew) - sum(Xnew)*sum(Ynew) )/( n*
    sum(Xnew.*Xnew) - ( sum(Xnew) )^2 )
13 lna = sum(Ynew)/n - b*sum(Xnew)/n
14 a = exp(lna)
15 disp(b, 'b = ')
16 disp(lna, 'lna = ')
17 disp(a, 'a = ')
18 printf('\n The power function equation obtained is \
    n y = %.4Gx^%.4G', a, b);

```

---

### Scilab code Exa 10.3 Fitting a Straight line using Regression

```

1 //Example No. 10_03
2 //Pg No. 332
3 clear ;close ;clc ;
4
5 time = 1:4
6 T = [ 70 83 100 124 ]
7 t = 6
8 Fx = exp(time/4)
9 n = length(Fx)
10 Y = T ;
11 b = ( n*sum(Fx.*Y) - sum(Fx)*sum(Y) )/( n*sum(Fx.*Fx)
    ) - ( sum(Fx))^2 )

```

```

12 a = sum(Y)/n - b*sum(Fx)/n
13 disp(b, 'b = ')
14 disp(a, 'a = ')
15 printf('The relationship between T and t is \n T = %
        .4G*e^(t/4) + %.4G \n', b, a)
16 deff('T = T(t)', 'T = b*exp(t/4) + a')
17 T_6 = T(6)
18
19 disp(T_6, 'The temperature at t = 6 is ')

```

---

### Scilab code Exa 10.4 Curve Fitting

```

1 //Example No. 10_04
2 //Curve Fitting
3 //Pg NO. 335
4 clear ; close ; clc ;
5
6 x = 1:4 ;
7 y = [6 11 18 27 ];
8 n = length(x) //Number of data points
9 m = 2+1          //Number of unknowns
10 disp('Using CA = B form , we get ')
11 for j = 1:m
12     for k = 1:m
13         C(j,k) = sum(x.^(j+k-2))
14     end
15     B(j) = sum( y.* ( x.^ ( j-1 ) ) )
16 end
17 disp(B, 'B = ', C, 'C = ')
18 A = inv(C)*B
19 disp(A, 'A = ')
20 printf('Therefore the least squares polynomial is \n
        y = %i + %i*x + %i*x^2 \n', A(1), A(2), A(3))

```

---

### Scilab code Exa 10.5 Plane Fitting

```
1 //Example No. 10_05
2 //Plane Fitting
3 //Pg No. 342
4 clear ; close ; clc ;
5
6 x = 1:4
7 z = 0:3
8 y = 12:6:30
9 n = length(x) //Number of data points
10 m = 3           //Number of unknowns
11 G = [ ones(1,n) ; x ; z]
12 H = G'
13 C = G*H
14 B = y*H
15 D = C\B,
16 disp(C,B)
17 disp(D)
18 mprintf('The regression plane is \n y = %i + %f*x
+ %iz ',D(1),D(2),D(3))
```

---

# Chapter 11

## Numerical Differentiation

Scilab code Exa 11.1 First order Forward Difference

```
1 //Example No. 11_01
2 //First order forward difference
3 //Pg No. 348
4 clear ;close ;clc ;
5
6 x = poly(0,"x");
7 deff('F = f(x)', 'F = x^2');
8 deff('DF = df(x,h)', 'DF = (f(x+h)-f(x))/h');
9 dfactual = derivat(f(x));
10 h = [0.2 ; 0.1 ; 0.05 ; 0.01 ]
11 for i = 1:4
12     y(i) = df(1,h(i));
13     err(i) = y(i) - horner(dfactual,1)
14 end
15 table = [h y err];
16 disp(table)
```

---

Scilab code Exa 11.2 Three Point Formula

```

1 //Example No. 11_02
2 //Three-Point Formula
3 //Pg No. 350
4 clear ;close ;clc ;
5
6 x = poly(0,"x");
7 deff('F = f(x)', 'F = x^2');
8 deff('DF = df(x,h)', 'DF = (f(x+h)-f(x-h))/(2*h)');
9 dfactual = derivat(f(x));
10 h = [0.2 ; 0.1 ; 0.05 ; 0.01 ]
11 for i = 1:4
12     y(i) = df(1,h(i));
13     err(i) = y(i) - horner(dfactual,1)
14 end
15 table = [h y err];
16 disp(table)

```

---

### Scilab code Exa 11.3 Error Analysis

```

1 //Example No. 11_03
2 //Pg No. 353
3 close ;clear ;clc ;
4
5 x = 0.45;
6 deff('F = f(x)', 'F = sin(x)');
7 deff('DF = df(x,h)', 'DF = (f(x+h) - f(x))/h');
8 dfactual = cos(x);
9 h = 0.01:0.005:0.04;
10 n = length(h);
11 for i = 1:n
12     y(i) = df(x,h(i))
13     err(i) = y(i) - dfactual ;
14 end
15 table = [ h' y err];
16 disp(table)

```

```

17 // scilab uses 16 significant digits so the bound for
    roundoff error is 0.5*10^(-16)
18 e = 0.5*10^(-16)
19 M2 = max(sin(x+h));
20 hopt = 2*sqrt(e/M2);
21 disp(hopt, 'hopt = ', M2, 'M2 = ')

```

---

#### Scilab code Exa 11.4 Approximate Second Derivative

```

1 //Example No. 11_04
2 //Approximate second derivative
3 //Pg No. 354
4 clear ; close ; clc ;
5
6 x = 0.75;
7 h = 0.01;
8 deff('F = f(x)', 'F = cos(x)');
9 deff('D2F = d2f(x,h)', 'D2F = ( f(x+h) - 2*f(x) + f(x-h) )/h^2 ');
10 y = d2f(0.75,0.01);
11 d2fexact = -cos(0.75)
12 err = d2fexact - y ;
13 disp(err, 'err = ', d2fexact, 'd2fexact = ', y, 'y = ')

```

---

#### Scilab code Exa 11.5 Differentiation of Tabulated Data

```

1 //Example No. 11_05
2 //Differentiation of tabulated data
3 //Pg No. 358
4 clear ; close ; clc ;
5
6 T = 5:9 ;
7 s = [10 14.5 19.5 25.5 32 ];

```

```

8 h = T(2)-T(1);
9 n = length(T)
10 function V = v(t)
11     if find(T == t) == 1 then
12         V = [ -3*s( find(T == t) ) + 4*s( find(T==(t+h)) ) - s( find( T == (t+2*h) ) ) ]/(2*h)
13             //Three point forward difference formula
14     elseif find(T == t) == n
15         V = [ 3*s( find(T == t) ) - 4*s( find(T==(t-h)) ) + s( find( T == (t-2*h) ) ) ]/(2*h)
16             //Backward difference formula
17     else
18         V = [ s( find(T == (t+h)) ) - s( find(T == (t-h)) ) ]/(2*h) //Central difference formula
19     end
20 endfunction
21
22 v_5 = v(5)
23 v_7 = v(7)
24 v_9 = v(9)
25
26 disp(v_9, 'v(9) = ', v_7, 'v(7) = ', v_5, 'v(5) = ')

```

---

### Scilab code Exa 11.6 Three Point Central Difference Formula

```

1 //Example No. 11_06
2 //Three Point Central Difference formula
3 //Pg No. 359
4 clear ;close ;clc ;
5
6 T = 5:9 ;
7 s = [10 14.5 19.5 25.5 32 ];
8 h = T(2)-T(1);

```

```

9 def( 'A = a( t ) ', 'A = [ s( find( T == ( t+h ) ) ) - 2*s
    ( find( T == t ) ) + s( find( T == ( t-h ) ) ) ]/h^2
    ')
10 a_7 = a(7)
11
12 disp(a_7, 'a(7) = ')

```

---

### Scilab code Exa 11.7 Second order Derivative

```

1 //Example No. 11_7
2 //Pg No. 359
3 clear ; close ; clc ;
4
5 h = 0.25 ;
6 //y'(x) = e^(x^2)
7 //y(0) = 0 , y(1) = 0
8 // y''(x) = y(x+h) - 2*y(x) + y(x-h)/h^2 = e^(x^2)
9 // ( y(x + 0.25) - 2*y(x) + y(x-0.25) )/0.0625 = e^(x
    ^2)
10 //y(x+0.25) - 2*y(x) + y(x - 0.25) = 0.0624*e^(x^2)
11 //y(0.5) - 2*y(0.25) + y(0) = 0.0665
12 //y(0.75) - 2*y(0.5) + y(0.25) = 0.0803
13 //y(1) - 2*y(0.75) + y(0.5) = 0.1097
14 //given y(0) = y(1) = 0
15 //
16 //0 + y2 - 2y1 = 0.06665
17 //y3 - 2*y2 + y1 = 0.0803
18 // -2*y3 + y2 + 0 = 0.1097
19 //Therefore
20 A = [0 1 -2 ; 1 -2 1 ; -2 1 0 ]
21 B = [ 0.06665 ; 0.0803 ; 0.1097 ]
22 C = A\B
23 mprintf('solving the above equations we get \n\n y1
    = y(0.25) = %f \n y2 = y(0.5) = %f \n y3 = y
    (0.75) = %f \n ',C(3),C(2),C(1))

```

---

### Scilab code Exa 11.8 Richardsons Extrapolation Technique

```
1 //Example No. 9_01
2 //Richardson's Extrapolation Technique
3 //Pg No. 362
4 clear ;close ;clc ;
5
6 x = -0.5:0.25:1.5
7 h = 0.5 ;
8 r = 1/2 ;
9
10 deff( 'F = f(x) ', 'F = exp(x) ');
11 deff( 'D = D(x,h) ', 'D = [ f(x + h) - f(x-h) ]/(2*h)
   ');
12 deff( 'df = df(x,h,r) ', 'df = [D(x,r*h) - r^2*D(x,h)
   ]/(1-r^2)');
13
14 df_05 = df(0.5,0.5,1/2);
15 disp(df_05,'richardsons technique - df(0.5) = ',D
   (0.5,0.25), 'D(rh) = D(0.25) = ',D(0.5,0.5), 'D
   (0.5) = ')
16 dfexact = derivative(f,0.5)
17 disp(dfexact,'Exact df(0.5) = ')
18 disp('The result by richardsons technique is much
   better than other results')
19
20 //r = 2
21 disp(df(0.5,0.5,2), 'df(x) = ',D(0.5,2*0.5), 'D(rh) =
   ', 'for r = 2')
```

---

# Chapter 12

## Numerical Integration

Scilab code Exa 12.1 Trapezoidal Rule

```
1 //Example No. 12_01
2 //Trapezoidal Rule
3 //Pg No. 373
4 clear ;close ;clc ;
5
6 x = poly(0,"x");
7 deff('F = f(x)', 'F = x^3 + 1');
8
9 //case(a)
10 a = 1;
11 b = 2 ;
12 h = b - a ;
13 It = (b-a)*(f(a)+f(b))/2
14 d2f = derivat(derivat(f(x)))
15 Ett = h^3*horner(d2f,2)/12
16 Iexact = intg(1,2,f)
17 Trueerror = It - Iexact
18 disp(Trueerror, 'True error = ', Iexact, 'Iexact = ',
      Ett, 'Ett = ', It, 'It = ', 'case(a)')
19 disp('Here Error bound is an overestimate of true
      error')
```

```

20
21 // case (b)
22 a = 1;
23 b = 1.5 ;
24 h = b - a ;
25 It = (b-a)*(f(a)+f(b))/2
26 Ett = h^3*horner(d2f,1.5)/12
27 Iexact = intg(1,1.5,f)
28 Trueerror = It - Iexact
29 disp(Trueerror,'True error = ',Iexact,'Iexact = ',
      Ett,'Ett = ',It,'It = ','case(b)')

```

---

### Scilab code Exa 12.2 Trapezoidal Rule

```

1 //Example No. 12_02
2 //Tapezoidal rule
3 //Pg No. 376
4 clear ;close ;clc ;
5
6 deff('F = f(x)', 'F = exp(x)');
7 a = -1 ;
8 b = 1 ;
9
10 // case (a)
11 n = 2
12 h = (b-a)/n
13 I = 0
14 for i = 1:n
15     I = I + f(a+(i-1)*h)+f(a+i*h);
16 end
17 I = h*I/2 ;
18 disp(I,'integral for case(a),Ia = ')
19
20 // case (b)
21 n = 4

```

```

22 h = (b-a)/n
23 I = 0
24 for i = 1:n
25     I = I + f(a+(i-1)*h)+f(a+i*h);
26 end
27 I = h*I/2 ;
28 Iexact = 2.35040
29 disp('n = 4 case is better than n = 2 case',Iexact,
      exact integral,Iexact = ',I,'integral for case(b
      ),Ib = ')

```

---

### Scilab code Exa 12.3 Simpons 1 by 3 rule

```

1 //Example No. 12_03
2 //Simpon's 1/3 rule
3 //Pg No. 381
4 clear ;close ;clc ;
5
6 funcprot(0) //To avoid warning message for defining
              function f(x) twice
7 //case(a)
8 deff('F = f(x)', 'F = exp(x)');
9 a = -1;
10 b = 1;
11 h = (b-a)/2
12 x1 = a+h
13 Is1 = h*( f(a) + f(b) + 4*f(x1) )/3
14 disp(Is1,'Integral for case(a) , Is1 = ',h,'h = ')
15
16 //case(b)
17 deff('F = f(x)', 'F = sqrt(sin(x))');
18 a = 0
19 b = %pi/2
20 h = (b-a)/2
21 x1 = a+h

```

```
22 Is1 = h*( f(a) + f(b) + 4*f(x1) )/3
23 disp(Is1, 'Integral for case(b), Is1 = ', h, 'h = ')
```

---

#### Scilab code Exa 12.4 Simpons 1 by 3 rule

```
1 //Example No. 12_04
2 //Simpon's 1/3 rule
3 //Pg No.382
4 clear ;close ;clc ;
5
6 def(f = f(x) , F = sqrt( sin(x) )) ;
7 x0 = 0 ;
8 xa = %pi/2 ;
9
10 //case(a) n = 4
11 n = 4 ;
12 h = (xa-x0)/n
13 I = 0
14 for i = 1:n/2
15     I = I + f(x0 + (2*i-2)*h) + 4*f(x0 + (2*i-1)*h)
16         + f(x0 + 2*i*h) ;
17 end
18 I = h*I/3
19 disp(I, 'Integral value for n = 4 is ', h, 'h = ')
20
21 //case(b) n = 6
22 n = 6
23 h = (xa-x0)/n
24 I = 0
25 for i = 1:n/2
26     I = I + f(x0 + (2*i-2)*h) + 4*f(x0 + (2*i-1)*h)
27         + f(x0 + 2*i*h) ;
28 end
29 I = h*I/3
30 disp(I, 'Integral value for n = 6 is ', h, 'h = ')
```

---

### Scilab code Exa 12.5 Simpsons 3 by 8 rule

```
1 //Example No. 12_05
2 //Simpson's 3/8 rule
3 //Pg No. 386
4 clear ;close ;clc ;
5
6 funcprot(0)
7 //case(a)
8 deff('F = f(x)', 'F = x^3 + 1');
9 a = 1 ;
10 b = 2 ;
11 h = (b-a)/3
12 x1 = a + h
13 x2 = a + 2*h
14 Is2 = 3*h*( f(a) + 3*f(x1) + 3*f(x2) + f(b) )/8 ;
15 disp(Is2, 'Integral of x^3 +1 from 1 to 2 is')
16 //case(b)
17 deff('F = f(x)', 'F = sqrt( sin(x) )');
18 a = 0 ;
19 b = %pi/2 ;
20 h = (b-a)/3
21 x1 = a + h
22 x2 = a + 2*h
23 Is2 = 3*h*( f(a) + 3*f(x1) + 3*f(x2) + f(b) )/8 ;
24 disp(Is2, 'Integral of sqrt( sin(x) ) from 0 to %pi/2
is ')
```

---

### Scilab code Exa 12.6 Booles Five Point Formula

```
1 //Example No. 12_06
```

```

2 //Booles's Five-Point formula
3 //Pg No. 387
4 clear ;close ;clc ;
5
6 def(f = f(x) , 'F = sqrt( sin(x) )')
7 x0 = 0;
8 xb = %pi/2 ;
9 n = 4 ;
10 h = (xb - x0)/n
11 Ib = 2*h*(7*f(x0) + 32*f(x0+h) + 12*f(x0 + 2*h) +
    32*f(x0+3*h) + 7*f(x0+4*h))/45;
12 disp(Ib , 'Ib = ')

```

---

### Scilab code Exa 12.7 Romberg Integration Formula

```

1 //Example No. 12_07
2 //Romberg Integration formula
3 //Pg No. 391
4 clear ;close ;clc ;
5
6 def(f = f(x) , 'F = 1/x');
7 //since we can't have (0,0) element in matrix we
    start with (1,1)
8 a = 1 ;
9 b = 2 ;
10 h = b-a ;
11 R(1,1) = h*(f(a)+f(b))/2
12 disp(R(1,1) , 'R(0,0) = ')
13 for i = 2:3
14     h(i) = (b-a)/2^(i-1)
15     s = 0
16     for k = 1:2^(i-2)
17         s = s + f(a + (2*k - 1)*h(i));
18     end
19     R(i,1) = R(i-1,1)/2 + h(i)*s;

```

```

20     printf (' \nR(%i,0) = %f \n' , i-1 , R(i,1))
21 end
22 for j = 2:3
23     for i = j:3
24         R(i,j) = (4^(j-1)*R(i,j-1) - R(i-1,j-1))
25             /(4^(j-1)-1);
26     printf (' \nR(%i,%i) = %f \n' , i-1 , j-1 , R(i,j))
27 end
28 end

```

---

### Scilab code Exa 12.8 Two Point Gauss Legefre Formula

```

1 //Example No. 12_08
2 //Two Point Gauss -Legedre formula
3 //Pg No. 397
4 clear ;close ;clc ;
5
6 deff( 'F = f(x)' , 'F = exp(x)' );
7 x1 = -1/sqrt(3)
8 x2 = 1/sqrt(3)
9 I = f(x1) + f(x2)
10 disp(I , 'I = ' , x2 , 'x2 = ' , x1 , 'x1 = ')

```

---

### Scilab code Exa 12.9 Gaussian Two Point Formula

```

1 //Example No. 12_09
2 //Gaussian two point formula
3 //Pg No. 398
4 clear ;close ;clc ;
5
6 a = -2 ;
7 b = 2 ;
8 deff( 'F = f(x)' , 'F = exp(-x/2)' )

```

```

9 A = (b-a)/2
10 B = (a+b)/2
11 C = (b-a)/2
12 deff( 'G = g(z) ', 'G = exp(-1*(A*z+B)/2) ')
13 w1 = 1 ;
14 w2 = 1 ;
15 z1 = -1/sqrt(3)
16 z2 = 1/sqrt(3)
17 Ig = C*( w1*g(z1) + w2*g(z2) )
18 printf('g(z) = exp(-(%f*z + %f)/2) \n C = %f \n Ig = %f \n ', A, B, C, Ig)

```

---

### Scilab code Exa 12.10 Gauss Legendre Three Point Formula

```

1 //Example No. 9_01
2 //Gauss-Legendre Three-point formula
3 //Pg No. 400
4 clear ; close ; clc ;
5
6 a = 2 ;
7 b = 4 ;
8 A = (b-a)/2
9 B = (b+a)/2
10 C = (b-a)/2
11 deff( 'G = g(z) ', 'G = (A*z + B)^4 + 1' )
12 w1 = 0.55556 ;
13 w2 = 0.88889 ;
14 w3 = 0.55556 ;
15 z1 = -0.77460;
16 z2 = 0 ;
17 z3 = 0.77460 ;
18 Ig = C*( w1*g(z1) + w2*g(z2) + w3*g(z3) )
19 printf('g(z) = (%f*z + %f)^4 + 1 \n C = %f \n Ig = %f \n ', A, B, C, Ig)

```

---

# Chapter 13

## Numerical Solution of Ordinary Differential Equations

Scilab code Exa 13.1 Taylor Method

```
1 //Example No. 13_01
2 //Taylor method
3 //Pg No. 414
4 clear ; close ; clc ;
5
6 deff( 'F = f(x,y) ', 'F = x^2 + y^2 ')
7 deff( 'D2Y = d2y(x,y) ', 'D2Y = 2*x + 2*y*f(x,y) ');
8 deff( 'D3Y = d3y(x,y) ', 'D3Y = 2 + 2*y*d2y(x,y) + 2*f(
    x,y)^2 ');
9 deff( 'Y = y(x) ', 'Y = 1 + f(0,1)*x + d2y(0,1)*x^2/2 +
    d3y(0,1)*x^3/6 ');
10 disp(y(0.25), 'y(0.25) = ')
11 disp(y(0.5), 'y(0.5) = ')
```

---

Scilab code Exa 13.2 Recursive Taylor Method

```

1 //Example No. 13_02
2 //Recursive Taylor Method
3 //Pg No. 415
4 clear ; close ; clc ;
5
6 deff( 'F = f(x,y)' , 'F = x^2 + y^2' )
7 deff( 'D2Y = d2y(x,y)' , 'D2Y = 2*x + 2*y*f(x,y)' );
8 deff( 'D3Y = d3y(x,y)' , 'D3Y = 2 + 2*y*d2y(x,y) + 2*f(
    x,y)^2' );
9 deff( 'D4Y = d4y(x,y)' , 'D4Y = 6*f(x,y)*d2y(x,y) + 2*y
    *d3y(x,y)' );
10 h = 0.2 ;
11 deff( 'Y = y(x,y)' , 'Y = y + f(x,y)*h + d2y(x,y)*h^2/2
    + d3y(x,y)*h^3/6 + d4y(x,y)*h^4/factorial(4)' );
12 x0 = 0;
13 y0 = 0 ;
14 for i = 1:2
15     y_(i) = y(x0,y0)
16     printf(' Iteration-%i\n',i)
17     dy(0) = %f\n
18     d2y(0) = %f\n
19     d3y(0) = %f\n
20     d4y(0) = %f\n
21     x0 = x0 + i*h
22     y0 = y_(i)
23     printf('y(0) = %f\n',y_(i))
24 end

```

---

### Scilab code Exa 13.3 Picards Method

```

1 //Example No. 13_3
2 //Picard's Method
3 //Pg No. 417
4 clear ; close ; clc ;
5 funcprot(0)
6 //y'(x) = x^2 + y^2, y(0) = 0
7 //y(1) = y0 + integral(x^2 + y0^2, x0, x)

```

```

8 //y(1) = x^3/3
9 //y(2) = 0 + integral(xY2 + y1^2,x0,x)
10 //      = integral(x^2 + x^6/9,0,x) = x^3/3 + x^7/63
11 // therefore y(x) = x^3/3 + x^7/63
12 deff('Y = y(x)', 'Y = x^3/3 + x^7/63')
13 disp(y(1), 'y(1) = ', y(0.2), 'y(0.2) = ', y(0.1), 'y
    (0.1) = ', 'for dy(x) = x^2 + y^2 the results are
    ')
14
15 //y'(x) = x*e^y, y(0) = 0
16 //y0 = 0, x0 = 0
17 //Y(1) = 0 + integral(x*e^0,0,x) = x^2/2
18 //y(2) = 0 + integral( x*e^( x^2/2 ) ,0,x) = e^(x
    ^2/2)-1
19 //therefore y(x) = e^(x^2/2) - 1
20 deff('Y = y(x)', 'Y = exp(x^2/2) - 1')
21 disp(y(1), 'y(1) = ', y(0.2), 'y(0.2) = ', y(0.1), 'y
    (0.1) = ', 'for dy(x) = x*e^y the results are ')

```

---

### Scilab code Exa 13.4 Eulers Method

```

1 //Example No. 13_04
2 //Euler's Method
3 //Pg No. 417
4 clear ; close ; clc ;
5
6 deff('DY = dy(x)', 'DY = 3*x^2 + 1')
7 x0 = 1
8 y(1) = 2 ;
9 //h = 0.5
10 h = 0.5
11 mprintf('for h = %f\n', h)
12 for i = 2 : 3
13     y(i) = y(i-1) + h*dy(x0+(i-2)*h)
14     mprintf('y(%f) = %f\n', x0+(i-1)*h, y(i))

```

```

15 end
16 //h = 0.25
17 h = 0.25
18 mprintf( '\nfor h = %f\n', h)
19 for i = 2 : 5
20     y(i) = y(i-1) + h*dy(x0+(i-2)*h)
21     mprintf( 'y(%f) = %f\n', x0+(i-1)*h, y(i))
22 end

```

---

### Scilab code Exa 13.5 Error Estimation in Eulers Method

```

1 //Example No. 13_05
2 //Error estimation in Euler's Method
3 //Pg No. 422
4 clear ; close ; clc ;
5
6 def('DY = dy(x)', 'DY = 3*x^2 + 1')
7 def('D2Y = d2y(x)', 'D2Y = 6*x')
8 def('D3Y = d3y(x)', 'D3Y = 6')
9 def('exacty = exacty(x)', 'exacty = x^3 + x')
10 x0 = 1
11 y(1) = 2
12 h = 0.5
13 for i = 2 : 3
14     x(i-1) = x0 + (i-1)*h
15     y(i) = y(i-1) + h*dy(x0+(i-2)*h)
16     mprintf( '\n Step %i \n x(%i) = %f\n y(%f) = %f\n',
17               i-1, i-1, x(i-1), x(i-1), y(i))
17     Et(i-1) = d2y(x0+(i-2)*h)*h^2/2 + d3y(x0+(i-2)*
18           h)*h^3/6
18     mprintf( '\n Et(%i) = %f\n', i-1, Et(i-1))
19     truey(i-1) = exacty(x0+(i-1)*h)
20     gerr(i-1) = truey(i-1) - y(i)
21 end
22

```

```

23 table = [x y(2:3) truey Et gerr]
24 disp(table, ' x Est y true y Et
    Globalerr')

```

---

### Scilab code Exa 13.6 Heuns Method

```

1 //Example No. 13_06
2 //Heun's Method
3 //Pg No. 427
4 clear ; close ;clc ;
5
6 deff( 'F = f(x,y)', 'F = 2*y/x' )
7 deff( 'exacty = exacty(x)', 'exacty = 2*x^2' )
8 x(1) = 1 ;
9 y(1) = 2 ;
10 h = 0.25 ;
11 //Euler's Method
12 disp('EULERS METHOD')
13 for i = 2:5
14     x(i) = x(i-1) + h ;
15     y(i) = y(i-1) + h*f(x(i-1),y(i-1));
16     mprintf('y(%f) = %f \n ',x(i),y(i))
17 end
18 eulery = y
19 //Heun's Method
20 disp('HEUNS METHOD')
21 for i = 2:5
22     m1 = f(x(i-1),y(i-1)) ;
23     ye(i) = y(i-1) + h*f(x(i-1),y(i-1));
24     m2 = f(x(i),ye(i)) ;
25     y(i) = y(i-1) + h*(m1 + m2)/2
26     mprintf('\nIteration %i \n m1 = %f\n ye(%f) = %f
        \n m2 = %f \n y(%f) = %f \n ',i-1,m1,x(i),ye(i)
        ,m2,x(i),y(i))
27 end

```

```
28 truey = exacty(x) ;
29 table = [x eulery y truey] ;
30 disp(table, ' x Eulers Heuns Analytical ')


---


```

### Scilab code Exa 13.7 Polygon Method

```
1 //Example No. 13_07
2 //Polygon Method
3 //Pg NO. 433
4 clear ; close ; clc ;
5 deff('F = f(x,y)', 'F = 2*y/x')
6 x(1) = 1 ;
7 y(1) = 2 ;
8 h = 0.25 ;
9 for i = 2:3
10     x(i) = x(i-1) + h ;
11     y(i) = y(i-1) + h*f( x(i-1)+ h/2 , y(i-1) + h
12         *f( x(i-1) , y(i-1) )/2 );
13     mprintf('y(%f) = %f \n ', x(i), y(i))
14 end


---


```

### Scilab code Exa 13.8 Classical Runge Kutta Method

```
1 //Example No. 13_08
2 //Classical Runge Kutta Method
3 //Pg No. 439
4 clear ; close ; clc ;
5
6 deff('F = f(x,y)', 'F = x^2 + y^2');
7 h = 0.2
8 x(1) = 0 ;
9 y(1) = 0 ;
```

```

10
11 for i = 1:2
12     m1 = f( x(i) , y(i) ) ;
13     m2 = f( x(i) + h/2 , y(i) + m1*h/2 ) ;
14     m3 = f( x(i) + h/2 , y(i) + m2*h/2 ) ;
15     m4 = f( x(i) + h , y(i) + m3*h ) ;
16     x(i+1) = x(i) + h ;
17     y(i+1) = y(i) + (m1 + 2*m2 + 2*m3 + m4)*h/6 ;
18
19     mprintf( '\nIteration - %i\n m1 = %f\n m2 = %f \n
20             m3 = %f \n m4 = %f \n y(%f) = %f \n', i, m1, m2
21             , m3, m4, x(i+1), y(i+1))
22 end

```

---

### Scilab code Exa 13.9 Optimum Step size

```

1 //Example No. 13_09
2 //Optimum step size
3 //Pg No. 444
4 clear ; close ; clc ;
5
6 x = 0.8 ;
7 h1 = 0.05 ;
8 y1 = 5.8410870 ;
9 h2 = 0.025 ;
10 y2 = 5.8479637 ;
11
12 //d = 4
13 h = ((h1^4 - h2^4)*10^(-4)/(2*(y2 - y1)))^(1/4)
14 disp(h, 'h = ', 'for four decimal places')
15
16 //d = 6
17 h = ((h1^4 - h2^4)*10^(-6)/(2*(y2 - y1)))^(1/4)
18 disp(h, 'h = ', 'for six decimal places')
19 disp('Note--We can use h = 0.01 for four decimal

```

places and  $h = 0.004$  for six decimal places ')

---

### Scilab code Exa 13.10 Milne Simpson Predictor Corrector Method

```
1 //Example No. 13_10
2 //Milne-Simpson Predictor-Corrector method
3 //Pg NO. 446
4 clear;close;clc;
5
6 deff( 'F = f(x,y)' , 'F = 2*y/x' )
7 x0 = 1 ;
8 y0 = 2 ;
9 h = 0.25 ;
10 //Assuming y1 ,y2 and y3(required for milne-simpson
    formula) are estimated using Fourth-
        order Runge
        kutta method
11 x1 = x0 + h
12 y1 = 3.13 ;
13 x2 = x1 + h
14 y2 = 4.5 ;
15 x3 = x2 + h
16 y3 = 6.13 ;
17 //Milne Predictor formula
18 yp4 = y0 + 4*h*(2*f(x1,y1) - f(x2,y2) + 2*f(x3,y3))
    /3
19 x4 = x3 + h
20 fp4 = f(x4,yp4) ;
21 disp(fp4,'fp4 = ',yp4,'yp4 = ')
22 //Simpson Corrector formula
23 yc4 = y2 + h*( f(x2,y2) + 4*f(x3,y3) + fp4)/3
24 f4 = f(x4,yc4)
25 disp(f4,'f4 = ',yc4,'yc4 = ')
26
27 yc4 = y2 + h*( f(x2,y2) + 4*f(x3,y3) + f4)/3
28 disp(yc4 , 'yc4 = ')
```

```
29 disp('Note— the exact solution is y(2) = 8')
```

---

### Scilab code Exa 13.11 Adams Bashforth Moulton Method

```
1 //Example No. 13_11
2 //Adams-Bashforth-Moulton Method
3 //Pg NO. 446
4 clear;close;clc;
5
6 deff('F = f(x,y)', 'F = 2*y/x')
7 x0 = 1 ;
8 y0 = 2 ;
9 h = 0.25 ;
10 x1 = x0 + h
11 y1 = 3.13 ;
12 x2 = x1 + h
13 y2 = 4.5 ;
14 x3 = x2 + h
15 y3 = 6.13 ;
16 //Adams Predictor formula
17 yp4 = y3 + h*(55*f(x3,y3) - 59*f(x2,y2) + 37*f(x1,y1)
18 ) - 9*f(x0,y0))/24
19 x4 = x3 + h
20 fp4 = f(x4,yp4)
21 disp(fp4,'fp4 = ',yp4,'yp4 = ','Adams Predictor
22 formula')
23 //Adams Corrector formula
24 yc4 = y3 + h*( f(x1,y1) - 5*f(x2,y2) + 19*f(x3,y3) +
25 9*fp4)/24
26 f4 = f(x4,yc4)
27 disp(f4,'f4 = ',yc4,'yc4 = ','Adams Corrector
28 formula')
29 yc4 = y3 + h*( f(x1,y1) - 5*f(x2,y2) + 19*f(x3,y3) +
30 9*f4)/24
```

```
27 disp(yc4 , 'refined-yc4 = ')
```

---

### Scilab code Exa 13.12 Milne Simpson Method Using Modifier

```
1 //Example No. 13_12
2 //Milne-Simpson Method using modifier
3 //Pg No. 453
4 clear ; close ; clc ;
5
6 deff( 'F = f(y)' , 'F = -y^2 ' )
7 x = [ 1 ; 1.2 ; 1.4 ; 1.6 ] ;
8 y = [ 1 ; 0.8333333 ; 0.7142857 ; 0.625 ] ;
9 h = 0.2 ;
10
11 for i = 1:2
12     yp = y(i) + 4*h*( 2*f( y(i+1) ) - f( y(i+2) ) +
13         2*f( y(i+3) ) )/3
14     fp = f(yp) ;
15     yc = y( i+2) + h*(f( y(i+2) ) + 4*f( y(i+3) ) +
16         fp )/3 ;
17     Etc = -(yc - yp)/29
18     y(i+4) = yc + Etc
19     mprintf( '\n y%ip = %f\n f%ip = %f \n y%ic = %f \
20     n Modifier Etc = %f \n Modified y%ic = %f \n',
21     ,i+3,yp,i+3,fp,i+3,yc,Etc,i+3,y(i+4))
22 end
23 exactanswer = 0.5 ;
24 err = exactanswer - y(6) ;
25 disp(err , 'error = ')
```

---

### Scilab code Exa 13.13 System of Differential Equations

```
1 //Example No. 13_13
```

```

2 //System of differential Equations
3 //Pg No. 455
4 clear ; close ; clc ;
5
6 deff( 'F1 = f1(x,y1,y2)', 'F1 = x + y1 + y2 ')
7 deff( 'F2 = f2(x,y1,y2)', 'F2 = 1 + y1 + y2 ')
8
9 x0 = 0 ;
10 y10 = 1 ;
11 y20 = -1 ;
12 h = 0.1 ;
13 m1(1) = f1( x0 , y10 , y20 )
14 m1(2) = f2( x0 , y10 , y20 )
15 m2(1) = f1( x0+h , y10 + h*m1(1) , y20 + h*m1(2) )
16 m2(2) = f2( x0+h , y10 + h*m1(1) , y20 + h*m1(2) )
17 m(1) = (m1(1) + m2(1))/2
18 m(2) = (m1(2) + m2(2))/2
19
20 y1_0_1 = y10 + h*m(1)
21 y2_0_1 = y20 + h*m(2)
22
23 mprintf('m1(1) = %f\n m1(2) = %f\n m2(1) = %f\n m2(2) = %f\n m(1) = %f\n m(2) = %f\n y1(0.1) = %f\n y2(0.1) = %f', m1(1), m1(2), m2(1), m2(2), m(1), m(2), y1_0_1, y2_0_1)

```

---

### Scilab code Exa 13.14 Higher Order Differential Equations

```

1 //Example No. 13_14
2 //Higher Order Differential Equations
3 //Pg No. 457
4 clear ; close ; clc ;
5
6 x0 = 0
7 y10 = 0

```

```

8  y20 = 1
9  h = 0.2
10 m1(1) = y20 ;
11 m1(2) = 6*x0 + 3*y10 - 2*y20
12 m2(1) = y20 + h*m1(2)
13 m2(2) = 6*(x0+h) + 3*(y10 + h*m1(1)) - 2*(y20 + h*m1
   (2))
14 m(1) = (m1(1) + m2(1))/2
15 m(2) = (m1(2) + m2(2))/2
16
17 y1_0_2 = y10 + h*m(1)
18 y2_0_2 = y20 + h*m(2)
19
20 mprintf('m1(1) = %f\n m1(2) = %f\n m2(1) = %f\n m2
   (2) = %f\n m(1) = %f\n m(2) = %f\n y1(0.1) = %f\n
   y2(0.1) = %f\n ',m1(1),m1(2),m2(1),m2(2),m(1),m
   (2),y1_0_2,y2_0_2)

```

---

# Chapter 14

## Boundary Value and Eigenvalue Problems

Scilab code Exa 14.1 Shooting Method

```
1 //Example No. 14_01
2 //Shooting Method
3 //Pg No. 467
4 clear ; close ; clc ;
5
6 function [B,y] = heun(f,x0,y0,z0,h,xf)
7     x(1) = x0 ;
8     y(1) = y0 ;
9     z(1) = z0 ;
10    n = (xf - x0)/h
11    for i = 1:n
12        m1(1) = z(i) ;
13        m1(2) = f(x(i),y(i))
14        m2(1) = z(i) + h*m1(2)
15        m2(2) = f(x(i)+h,y(i)+h*m1(1))
16        m(1) = (m1(1) + m2(1))/2
17        m(2) = ( m1(2) + m2(2) )/2
18        x(i+1) = x(i) + h
19        y(i+1) = y(i) + h*m(1)
```

```

20           z(i+1) = z(i) + h*m(2)
21       end
22   B = y(n+1)
23 endfunction
24
25 def f('F = f(x,y)', 'F = 6*x')
26 x0 = 1 ;
27 y0 = 2 ;
28 h = 0.5 ;
29 z0 = 2
30 M1 = z0
31 xf = 2
32 B = 9
33 [B1,y] = heun(f,x0,y0,z0,h,xf)
34 disp(B1,'B1 = ')
35 if B1 ~= B then
36     disp('Since B1 is less than B , let z(1) = y(1)
            = 4*(M2)')
37     z0 = 4
38     M2 = z0
39     [B2,y] = heun(f,x0,y0,z0,h,xf)
40     disp(B2,'B2 = ')
41     if B2 ~= B then
42         disp('Since B2 is larger than B , let us have
                third estimate of z(1) = M3 ')
43         M3 = M2 - (B2 - B)*(M2 - M1)/(B2 - B1)
44         z0 = M3
45         [B3,y] = heun(f,x0,y0,z0,h,xf)
46         disp(y,'The solution is ',B3,'B3 = ')
47     end
48 end

```

---

### Scilab code Exa 14.2 Finite Difference Method

1 //Example No. 14\_02

```

2 //Finite Difference Method
3 //Pg No. 470
4 clear ; close ; clc ;
5
6 deff( 'D2Y = d2y(x)', 'D2Y = exp(x^2)')
7 x_1 = 0;
8 y_0 = 0 ;
9 y_1 = 0 ;
10 h = 0.25
11 xf = 1
12 n = (xf-x_1)/h
13 for i = 1:n-1
14     A(i,:) = [1 -2 1]
15     B(i,1) = exp((x_1 + i*h)^2)*h^2
16 end
17 A(1,1) = 0 ; //since we know y0 and y4
18 A(3,3) = 0 ;
19 A(1,1:3) = [ A(1,2:3) 0] //rearranging terms
20 A(3,1:3) = [ 0 A(3,1:2)]
21 C = A\B //Solution of Equations
22 mprintf( '\n The solution is \n y1 = y(0.25) = %f \n
              y2 = y(0.5) = %f \n y3 = y(0.75) = %f \n ',C(1),
              C(2),C(3))

```

---

### Scilab code Exa 14.3 Eigen Vectors

```

1 //Example No. 14_03
2 //Eigen Vectors
3 //Pg No. 473
4 clear ; close ; clc ;
5
6 A = [8 -4 ; 2 2] ;
7 lamd = poly(0,'lamd')
8 p = det(A - lamd*eye())
9 root = roots(p)

```

```

10 mprintf(' \n The roots are \n lamda1 = %f \n lamda2 =
           %f \n ',root(1),root(2))
11 A1 = A - root(1)*eye()
12 X1 = [-1*A1(1,2)/A1(1,1) ; 1]
13 disp(X1,'X1 = ')
14 A2 = A - root(2)*eye()
15 X2 = [-1*A2(1,2)/A2(1,1) ; 1]
16 disp(X2,'X2 = ')

```

---

#### Scilab code Exa 14.4 Fadeev Leverrier Method

```

1 //Example No. 14_04
2 //Fadeev – Leverrier method
3 //Pg No. 474
4 clear ; close ; clc ;
5
6 A = [ -1 0 0 ; 1 -2 3 ; 0 2 -3 ]
7 [r,c] = size(A)
8 A1 = A
9 p(1) = trace(A1)
10 for i = 2:r
11     A1 = A*( A1 - p(i-1)*eye()) 
12     p(i) = trace(A1)/i
13     mprintf(' \n A%i = ',i)
14     disp(A1)
15     mprintf(' \n p%i = %f \n ',i,p(i))
16 end
17 x = poly(0,'x');
18 p = p($:-1:1)
19 polynomial = poly([-p ; 1], 'x', 'coeff')
20 disp(polynomial,'Charateristic polynomial is ')

```

---

#### Scilab code Exa 14.5 Eigen Vectors

```

1 //Example No. 14_05
2 //Eigen Vectors
3 //Pg No. 476
4
5 clear ; close ; clc ;
6
7 A = [ -1 0 0 ; 1 -2 3 ; 0 2 -3]
8 [eectors,evalues] = spec(A)
9 for i = 1:3
10    mprintf ('\n Eigen vector - %i \n for lamda%i =
           %f \n X%i = ',i,i,evalues(i,i),i)
11    eectors(:,i) = eectors(:,i)/eectors(2,i)
12    disp(eectors(:,i))
13 end

```

---

### Scilab code Exa 14.6 Power Method

```

1 //Example No. 14_06
2 //Power method
3 //Pg No. 478
4 clear ; close ; clc ;
5
6 A = [ 1 2 0 ; 2 1 0 ; 0 0 -1 ]
7 X(:,1) = [0 ; 1 ; 0]
8 for i = 1:7
9    Y(:,i) = A*X(:,i)
10   X(:,i+1) = Y(:,i)/max(Y(:,i))
11 end
12 disp(' 0      1      2      3      4
           5      6      7 ','')
13 disp('Iterations')
14 disp(X, 'X = ', [%nan ; %nan ; %nan] Y ], 'Y = ')

```

---

# Chapter 15

## Solution of Partial Differential Equations

Scilab code Exa 15.1 Elliptic Equations

```
1 //Example No. 15_01
2 //Elliptic Equations
3 //Pg No. 488
4 clear ; close ; clc ;
5
6 l = 15
7 h = 5
8 n = 1 + 15/5
9 f(1,1:4) = 100 ;
10 f(1:4,1) = 100 ;
11 f(4,1:4) = 0 ;
12 f(1:4,4) = 0 ;
13
14 //At point 1 : f2 + f3 - 4f1 + 100 + 100 = 0
15 //At point 2 : f1 + f4 - 4f2 + 100 + 0 = 0
16 //At point 3 : f1 + f4 - 4f3 + 100 + 0 = 0
17 //At point 4 : f2 + f3 - 4f4 + 0 + 0 = 0
18 //
19 //Final Equations are
```

```

20 // -4f1 + f2 + f3 + 0 = -200
21 // f1 - 4f2 + 0 + f4 = -100
22 // f1 + 0 - 4f3 + f4 = -100
23 // 0 + f2 + f3 - 4f4 = 0
24 A = [ -4 1 1 0 ; 1 -4 0 1 ; 1 0 -4 1 ; 0 1 1 -4 ]
25 B = [-200 ; -100 ; -100 ; 0]
26 C = A\B
27 mprintf( '\n The solution of the system is \n f1 = %i
           \n f2 = %i \n f3 = %i \n f4 = %f ',C(1),C(2),
           C(3),C(4))

```

---

### Scilab code Exa 15.2 Liebmans Iterative Method

```

1 //Example No. 15_02
2 //Liebmann's Iterative method
3 //Pg No. 489
4 clear ; close ; clc ;
5
6 f(1,1:4) = 100 ;
7 f(1:4,1) = 100 ;
8 f(4,1:4) = 0 ;
9 f(1:4,4) = 0 ;
10 f(3,3) = 0
11 for n = 1:5
12     for i = 2:3
13         for j = 2:3
14             if n == 1 & i == 2 & j == 2 then
15                 f(i,j) = ( f(i+1,j+1) + f(i-1,j-1) +
16                             f(i-1,j+1) + f(i+1,j-1) )/4
17             else
18                 f(i,j) = ( f(i+1,j) + f(i-1,j) + f(i,
19                               ,j+1) + f(i,j-1) )/4
20             end
21         end
22     end
23 end

```

```

21      A(2:5,n) = [f(2,2);f(2,3) ; f(3,2) ; f(3,3) ]
22 end
23 A(1,1:5) = 0:4
24 disp(A,'First row of below matrix represents
iteration number')

```

---

### Scilab code Exa 15.3 Poissons Equation

```

1 //Example No. 15_03
2 //Poisson's Equation
3 //Pg No. 490
4 clear ; close ; clc ;
5
6 //D2f = 2*x^2 * y^2
7 // f = 0
8 // h = 1
9 //Point 1 : 0 + 0 + f2 + f3 - 4f1 = 2(1)^2 * 2^2
10 // f2 + f3 - 4f1 = 8
11 //Point 2 : 0 + 0 + f1 + f4 - 4f2 = 2*(2)^2*2^2
12 // f1 - 4f2 = f4 = 32
13 //Point 3 : 0 + 0 + f1 + f4 - 4f4 = 2*(1^2)*1^2
14 // f1 - 4f3 + f4 = 2
15 //Point 4 : 0 + 0 + f2 + f3 - 4f4 = 2* 2^2 * 1^2
16 // f2 + f3 - 4f4 = 8
17 //Rearranging the equations
18 // -4f1 + f2 + f3 = 8
19 // f1 - 4f2 + f4 = 32
20 // f1 - 4f3 + f4 = 2
21 // f2 + f3 - 4f4 = 8
22 A = [ -4 1 1 0 ; 1 -4 0 1 ; 1 0 -4 1 ; 0 1 1 -4]
23 B = [ 8 ; 32 ; 2 ; 8 ]
24 C = A\B ;
25 mprintf('The solution is \n f1 = %f \n f2 = %f \n f3
= %f \n f4 = %f \n ', C(1),C(2),C(3),C(4))

```

---

### Scilab code Exa 15.4 Gauss Siedel Iteration

```
1 //Example No. 15_04
2 //Gauss–Seidel Iteration
3 //Pg No. 491
4 clear ; close ; clc ;
5
6 f2 = 0
7 f3 = 0
8 for i = 1:4
9     f1 = (f2 + f3 - 8)/4
10    f4 = f1
11    f2 = (f1 + f4 -32)/4
12    f3 = (f1 + f4 - 2)/4
13    mprintf( '\nIteration %i\n f1 = %f,      f2 = %f,
14          f3 = %f,      f4 = %f\n',i,f1,f2,f3,f4)
14 end
```

---

### Scilab code Exa 15.5 Initial Value Problems

```
1 //Example No. 15_05
2 //Initial Value Problems
3 //Pg No. 494
4 clear ; close ; clc ;
5
6 h = 1 ;
7 k = 2 ;
8 tau = h^2/(2*k)
9 for i = 2:4
10    f(1,i) = 50*( 4 - (i-1) )
11 end
12 f(1:7,1) = 0 ;
```

```

13 f(1:7,5) = 0 ;
14 for j = 1:6
15     for i = 2:4
16         f(j+1,i) = ( f(j,i-1) + f(j,i+1) )/2
17     end
18 end
19 disp(f,'The final results are ')

```

---

### Scilab code Exa 15.6 Crank Nicholson Implicit Method

```

1 //Example No. 15_06
2 //Crank–Nicholson Implicit Method
3 //Pg No. 497
4 clear ; close ; clc ;
5
6 h = 1 ;
7 k = 2 ;
8 tau = h^2/(2*k)
9 for i = 2:4
10    f(1,i) = 50*( 4 - (i-1) )
11 end
12 f(1:5,1) = 0 ;
13 f(1:5,5) = 0 ;
14 A = [4 -1 0 ; -1 4 -1 ; 0 -1 4]
15 for j = 1:4
16    for i = 2:4
17        B(i-1,1) = f(j,i-1) + f(j,i+1)
18    end
19    C = A\B
20    f(j+1,2) = C(1)
21    f(j+1,3) = C(2)
22    f(j+1,4) = C(3)
23 end
24 disp(f,'The final solution using crank nicholson
implicit method is ')

```

---

### Scilab code Exa 15.7 Hyperbolic Equations

```
1 //Example No. 15_07
2 //Hyperbolic Equations
3 //Pg No. 500
4 clear ; close ;clc ;
5
6 h = 1
7 Tbyp = 4
8 tau = sqrt(h^2 /4)
9 r = 1+(2.5 - 0)/tau
10 c = 1+(5 - 0)/h
11 for i = 2:c-1
12     f(1,i) = (i-1)*(5 - (i-1) )
13 end
14 f(1:r,1) = 0
15 f(1:r,c) = 0
16 for i = 2:c-1
17     g(i) = 0
18     f(2,i) = (f(1,i+1) + f(1,i-1))/2 + tau*g(i)
19 end
20 for j = 2:r-1
21     for i = 2:c-1
22         f(j+1,i) = -f(j-1,i) + f(j,i+1) + f(j,i-1)
23     end
24 end
25 disp(f, 'The values estimated are ')
```

---