Optimisation Problems

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1. Linear Programming
   Maximize
   $3x_1 + x_2 + 3x_3$
   for
   $2x_1 + x_2 + x_3 \leq 2$
   $x_1 + 2x_2 + 3x_3 \leq 5$
   $2x_1 + 2x_2 + x_3 \leq 6$
   $x1, x2, x3 \geq 0$

   SOLUTION

```
// This is a linear programming with linear constraints.This programm uses karmarkar ,an inbuilt function.It can also be solved in scilab using linpro function (linpro is in Quapro toolbox under Optimization in ATOMS,it can installed by command atomsInstall(quapro))

mode(0);
A = [
      2 1 1
      1 2 3
      2 2 1]; // the matrix of linear inequality constraints.
b = [2;5;6];// the right-hand side of linear inequality constraints.
c = [3;1;3];// the linear part of the objective function.
lb = [0;0;0]; // the lower bounds
[xopt,fopt,exitflag,iter,yopt] =karmarkar ([],[],c,[],[],[],[],[],A,b,lb) ;//Solves a linear optimization problem
disp(xopt(1),"x1=")
disp(xopt(2),"x2=")
disp(xopt(3),"x3=")
disp(fopt," the objective function value at optimum
```

```
")
disp(iter,"the number of iterations=")
disp(yopt,"a struct containing the dual solution. The structure yopt has four fields : ineqlin, eqlin, upper, lower.")
disp(yopt.ineqlin,"yopt.ineqlin=")
disp(yopt.lower,"yopt.lower=")
browsevar() //// browsevar can show all variables. browsevar can be costumized to show all or some type of variables. It's also possible to exclude variable names.
```

2. Quadratic Programming:
   Minimize
   $f(x) = \frac{1}{2}x^T \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} -2 \\ -2 \end{bmatrix}^T x$
   for
   $x_1 + x_2 \geq 2 + \sqrt{2}$
   $-x_1 + x_2 \geq -2$

   SOLUTION

```
// This is a linear quadrtatic programm with linear constraints.This programm uses qp_solve ,an inbuilt function.It can also be solved in scilab using quapro function (quapro is in Quapro toolbox under Optimization in ATOMS,it can installed by command atomsInstall(quapro))

mode(0)
Q = [2 0;0 2]; //real positive definite symmetric matrix (dimension n x n).
p=[-2;-2]; //real (column) vector (dimension n)
C1=[1 1;-1 1];
b1=[(2+2^1/2);-2];
XL=[0;0]; // column vector of lower-bounds
XU=[1e10;1e10]; //column vector of upper-bounds
```

```
me=2; //number of equality constraints
[xopt,iact,iter,fopt] = qp_solve(Q,p,C1,b1,me) //qp_solve is a linear quadratic programming solver
disp(xopt(1),"x1=")
disp(xopt(2),"x2=")
disp(fopt," the objective function value at optimum=")
disp(iter(1),"The number of main iterations=")
disp(iter(2),"How many constraints were deleted after they became active=")
disp(iact,"vector, indicator of active constraints. The first non zero entries give the index of the active constraints")
browsevar() // browsevar can show all variables. browsevar can be costumized to show all or some type of variables. It's also possible to exclude variable names.
```

3. Non-Linear Programming:
   Minimize
   $(x_1 - 2)^2 + (x_2 - 1)^2$
   for
   $g1 : -x_2 + x_1^2 \leq 0$
   $x_1 + x_2 \leq 2$
   $x_2 \geq 0$

   Find Contour Surface Plot and Unconstrained Minima of the following problems

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4. $f(x) = x_1^2 + x_2^2 + x_1 x_2$

   SOLUTION

```
//This programm computes the unconstrained minimimum of given function using an inbuilt fminsearch function.
```

```
mode(0)
function y=f(x)
        y=x(1)^2 +x(2)^2+x(1)*x(2);
    endfunction

    opt = optimset ( "TolX" , 1.e-2 ); // Optimset functionmanages the "options" data structure, which is a struct with a set of fields (for example, "MaxFunEvals", "MaxIter", etc...)

    [x,fval,exitflag,output] = fminsearch ( f , [-1.2 1]); // computes the unconstrained minimimum of given function with the Nelder-Mead algorithm.

    disp(x,"The minimum=")
    disp(fval,"The minimum function value=")
    exitflag //The flag associated with exist status of the algorithm.The help file for fminsearch will give details of different values of exitflag.
    output //A struct which stores detailed information about the exit of the algorithm.
    browsevar()
    //******** Level curves of a surface on a 2D plot *************
    deff('[w]=f(x1,x2)','w=x1^2+x2^2+x1*x2')
  x1=[0:0.25:6];x2=[0:0.25:6];z=feval(x1,x2,f);
  contour2d(x1,x2,z,10)
  xtitle('Contour plot of w=x1^2+x2^2+x1*x2')
```

5. $(x_2 - x_1)^2 + 8x_1x_2 - x_1 + x_2 + 3$
   SOLUTION

```
//This programm computes the unconstrained minimimum of given function using an inbuilt
```

```
fminsearch function.
mode(0)
function y=f(x)
        y = -(((x(2))-(x(1)))^2+8*(x(1))*(x(2))+(x(1))*(x(2)));
    endfunction
    opt = optimset ( "TolX" , 1.e-2 );// Optimset functionmanages the "options" data structure, which is a struct with a set of fields (for example, "MaxFunEvals", "MaxIter", etc...)
    [x,fval,exitflag,output] = fminsearch ( f , [-1.2 1]); // computes the unconstrained minimimum of given function with the Nelder-Mead algorithm.
   disp(x,"The minimum=")
    disp(fval,"The minimum function value=")
    exitflag //The flag associated with exist status of the algorithm.The help file for fminsearch will give details of different values of exitflag.
    output //A struct which stores detailed information about the exit of the algorithm.
    browsevar()
    //******** Level curves of a surface on a 2D plot *************
    deff('[w]=f(x1,x2)','w=(x2-x1)^2+8*x1*x2-x1+x2+3')
  x1=[0:0.25:6];x2=[0:0.25:6];z=feval(x1,x2,f);
  contour2d(x1,x2,z,10)
  xtitle('Contour plot of w=(x2-x1)^2+8*x1*x2-x1+x2+3','x1','x2')
```

6. $(x_1^2 - 1.5x_1x_2 + 2x_2^2)x_1^2$

SOLUTION

```
//This programm computes the unconstrained minimimum of given function using an inbuilt fminsearch function.
mode(0)
function y=f(x)
        y = -((x(1))^2-1.5*(x(1))*(x(2)))
        ;
    endfunction

    opt = optimset ( "TolX" , 1.e-2 );// Optimset functionmanages the "options" data structure, which is a struct with a set of fields (for example, "MaxFunEvals", "MaxIter", etc...)
    [x,fval,exitflag,output] = fminsearch ( f , [-1.2 1]);// computes the unconstrained minimimum of given function with the Nelder-Mead algorithm.
 disp(x,"The minimum=")
    disp(fval,"The minimum function value=")
    exitflag //The flag associated with exist status of the algorithm.The help file for fminsearch will give details of different values of exitflag.
    output //A struct which stores detailed information about the exit of the algorithm.
    browsevar()
    //******** Level curves of a surface on a 2D plot *************

    deff('[w]=f(x1,x2)','w=(((x1)^2)*(x1)^2-1.5*(x2)*(x2)*(x1)^2+2*(x2)^2)*(x1)^2') //an inline function
   x1=[0:0.25:6];x2=[0:0.25:6];z=feval(x1,x2,f); // multiple evaluation

   contour2d(x1,x2,z,10)
```

```
xtitle('Contour plot of w=(((x1)^2)-1.5*(x2)*(x2)+2*(x2)^2)(x1)^2','x1','x2')
```

GRAPHICAL COMPUTATION OF Q6

```
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[xopt,fopt,exitflag,iter,yopt] =karmarkar ([],[],c,[],[],[],[],[],A,b,lb) ;//Solves a linear optimization problem
disp(xopt(1),"x1=")
disp(xopt(2),"x2=")
disp(xopt(3),"x3=")
disp(fopt," the objective function value at optimum ")
disp(iter,"the number of iterations=")
disp(yopt,"a struct containing the dual solution. The structure yopt has four fields : ineqlin, eqlin, upper, lower.")
disp(yopt.ineqlin,"yopt.ineqlin=")
disp(yopt.lower,"yopt.lower=")
```

```
browsevar() //// browsevar can show all variables.
    browsevar can be costumized to show all or some
    type of variables. It's also possible to exclude
     variable names.
```