FOSSEE Optimization Toolbox Workshop
Workshop Material

December 2016

1 Scilab Basics

Scilab is a numerical computation package which has a wide array of functions and toolboxes. It is an open-source alternative to other mathematical packages like Matlab and Mathematica. Everything in Scilab is stored as a matrix. Even a real number is stored as a 1×1 matrix as given in the following snippet.

```
-->a=5
a  =


    1.
-->size(a)
 ans  =


    1.    1.
```

Matrices in general are combinations of rows and columns. A one-dimensional matrix is called as a vector. A row vector consists of multiple elements stored is a row. A column vector has multiple elements stored is a column.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

2×2 matrix   Row vector   column vector

The above vectors can be given as input into Scilab as follows:

```
-->a=[1, 2 ;3, 4]
 a  =


   1.    2.
   3.    4.
-->b=[1,2,3,4,5]
 b  =


   1.    2.    3.    4.    5.
-->c=[1;2;3;4;5]
 c  =
   1.
   2.
   3.
   4.
   5.
```

Scilab provides an array of commands,functions and toolboxes for various operations. Some basic matrix operations in Scilab are given below:

```
-->d=[b;b] //joining two matrices by row
 d  =
   1.    2.    3.    4.    5.
   1.    2.    3.    4.    5.
-->a*a //matrix multiplication
 ans  =
   7.     10.
   15.    22.
-->a.*a //element-wise multiplication
 ans  =
   1.    4.
   9.    16.
-->size(a) //returns the size of the matrix
 ans  =
   2.    2.
-->a' //transposes the matrix
 ans  =
   1.    3.
   2.    4.
-->zeros(2,5) //returns a zero matrix of dimensions 2x5
 ans  =
```

```
   0.   0.   0.   0.   0.
   0.   0.   0.   0.   0.
-->eye(2,2) //returns an identity matrix of dimensions 2x2
 ans  =

   1.   0.
   0.   1.
-->1:10 // returns a matrix of series from 1 to 10 with interval of 1
 ans  =

   1.   2.   3.   4.   5.   6.   7.   8.   9.   10.
-->1:2:10 //returns a matrix of series from 1 to 9 with interval of 2
 ans  =

   1.   3.   5.   7.   9.
```

---

2 Linear Programming

1. (a) Solve the following mathematical model in *linprog*:

$$\begin{aligned} \text{Minimize } & x_1 - 2x_2 + x_3 \\ \text{subject to } & -x_1 - x_2 - x3 \leq -5 \\ & 1.5x_2 - x_1 \leq 7 \\ & x_1 - x_3 \leq -10 \end{aligned} \qquad \begin{aligned} c &= \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \\ A &= \begin{bmatrix} -1 & -1 & -1 \\ -1 & 1.5 & 0 \\ 1 & 0 & -1 \end{bmatrix} \\ b &= \begin{bmatrix} -5 \\ 7 \\ -10 \end{bmatrix} \end{aligned} \tag{1}$$

   (b) Solve the above problem with the following additional bounds.

$$\begin{aligned} & x_1, x_2, x_3 \geq 0 \\ & x_1, x_2, x_3 \leq 10 \end{aligned} \qquad \begin{aligned} lb &= \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \\ ub &= \begin{bmatrix} 10 & 10 & 10 \end{bmatrix} \end{aligned} \tag{2}$$

(c) Solve the above problem with the following equality constraints.

$$x_1 = 2 \qquad \begin{aligned} Aeq &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ beq &= \begin{bmatrix} 4 \end{bmatrix} \end{aligned} \tag{3}$$

2. Solve the following mathematical model in *linprog*:

$$\begin{aligned} \text{Minimize } & \sum_{i=0}^{10} i.x_i \\ \text{subject to } & x_i - x_{i+1} \leq 0 \qquad \forall\, i = 1, 2 \cdots 9 \\ & \sum_{i=0}^{10} x_i = 30 \\ & 2 \leq x_i \leq 5 \end{aligned} \tag{4}$$

3. Formulate and solve the following mathematical model in *linprog*:

$$\begin{aligned} \text{Maximize } & x_1 + 2x_2 \\ \text{subject to } & x_1 - x_2 \leq 2 \\ & x_1 \geq 1 \\ & x_2 = 5 \end{aligned} \tag{5}$$

Note: Convert the model to a form which can be understood by linprog.

4. Solve problem 1(a) with integer constraints on variables x_2 and x_3.
Note: Observe the difference between 1(a) and this solution

5. Solve the following mathematical model using *symphonymat*.

$$\text{w} = \begin{pmatrix} 26 & 63 & 40 & 91 & 4 & 48 & 26 & 41 & 28 & 12 \end{pmatrix}$$

$$x_i = \begin{cases} 1 & \text{if i is in group 1} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} & \text{Minimize } \sum_{i=0}^{10} w_i.x_i \\ \text{subject to } \quad & \sum_{i=0}^{10} w_i.x_i \geq \frac{1}{2} \sum_{i=0}^{10} w_i \end{aligned} \tag{6}$$

Note: The above formulation is the mathematical model for a Number Partitioning Problem.A number partitioning problem is defined as follows: Partition a given set of positive integers into two mutually exclusive subsets such that the difference of the sum of either subset is minimized.

3 Nonlinear Programming

1. (a) Find the minimum value of the following equation using *fminunc*.

$$\text{Min} \quad x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + 10x_1 + 9x_3 \tag{7}$$

   (b) Solve the above problem with the following bounds *fminbnd*:

$$x_1, x_2, x_3, x_4, x_5 \geq -3$$
$$x_1, x_2, x_3, x_4, x_5 \leq 3$$

   (c) Solve the above problem with the following constraints *fmincon*.

$$x_2 - x_1 \leq -4$$
$$x_1 + x_5 \leq 10$$

2. (a) Solve the following optimization problem *fminunc* where X,Y and Z are the decision variables:

$$\text{Minimize } \sum_{i=0}^{10}(X - x_i)^2 + \sum_{i=0}^{10}(Y - y_i)^2 + \sum_{i=0}^{10}(Z - z_i)^2 \tag{8}$$

   where

$$x = \Big(10, 5, 7, 1, 0, 5, 8, 6, 2, 9\Big)$$
$$y = \Big(6, 9.5, 8, 1, 0, 2.5, 1, 0, 2, 6\Big)$$
$$z = \Big(2, 1, 0, 10, 0, 2.5, 1, 5, 2, 6\Big)$$

   Note: The above problem is the mathematical model to find the centroid of a 10 points. In this case, sets x,y and z forms the coordinates of the points.

   (b) What is the best possible solution if the point has to lie inside the box defined by:

$$\begin{aligned} 4 \leq X \leq 6 \\ 5 \leq Y \leq 7 \\ 4.5 \leq Z \leq 7 \end{aligned} \tag{9}$$

   (c) Solve the above problem using *qpipopt* along with the following constraint.

$$X + Y + Z \leq 15 \tag{10}$$

4 Assignments

1. SA company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B. At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours. The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

   (a) Formulate the problem of deciding how much of each product to make in the current week as a linear program.

   (b) Solve this linear program using *linprog*.

2. Find the optimal values for the following equations using *fmincon*:

$$\begin{aligned} \text{Min} \quad & x_1^2 + x_2^4 \\ \text{Subject to} \quad & x_1 + x_2 \geq 5 \\ & x_1 \leq 10 \\ & x_1, x_2 \geq 0 \end{aligned} \tag{11}$$