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NUMERICAL SOLUTION FOR THE DESIGN OF MINIMUM LENGTH SUPERSONIC NOZZLE

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ABSTRACT

Minimum length of the supersonic nozzle has been calculated for the optimum Mach number at the nozzle exit with uniform flow at both converging and diverging section of the nozzle. The calculation has been carried out based on the method of characteristics. Numerical solution is established for the two-dimensional, steady, in viscid, irrotational and supersonic flow. It is rational to assume the flow holds the consistency in the converging section and, thereby, an arbitrary shape is assumed for the converging section of the converging-diverging nozzle. The design considerations are concentrated at the diverging section.

Keywords: supersonic, nozzle design, numerical solution.

1. INTRODUCTION

In a convergent-divergent nozzle, in subsonic flow, flow area decrease results in acceleration of the gas. The flow area converges until the minimum area, namely throat area, is reached. In throat, the flow travels at sonic speed. In downstream of the throat, the diverging section can speed up the gas to the supersonic limit. The nozzle allows the pressure developed by the combustion chamber or the blower to increase the thrust at outlet by accelerating the exhaust or outlet gas to a high supersonic speed. The velocity limit of a convergent-divergent nozzle is governed by the area ratio. On the other hand, the area ratio is determined by the design ambient pressure into which the nozzle discharges. High thrust efficiency can be achieved by the painstaking design of the nozzle contour. In a vacuum, the maximum theoretical performance can be experienced by an ideal nozzle with infinite exit area which will allow expanding the gases to zero pressure, hence, attaining the maximum velocity. If ambient pressure is excessive, the engine exhaust flow will be separated from the nozzle wall and large nozzle side loads may develop due to the uneven separation of the exhaust jet. These loads must be included in the nozzle structural design process.

Gas properties at nozzle area ratio can be obtained from a property of the gas called the ratio of specific heat capacities. The ratio, for a perfect gas, remains constant throughout the expansion process. In actual, as the gas expands, the ratio shifts due to the changes in temperature and the chemical composition. Maximum thrust is obtained if the gas composition is in chemical equilibrium throughout the entire nozzle expansion process.

In a conventional bell nozzle, the gas flowing from the combustion chamber and through the nozzle throat must be turned away from the nozzle axis in order to accelerate or expand the flow. Afterwards, the flow must be turned back parallel to the nozzle axis to maximize nozzle efficiency.

Now it is apparent that there are three types of losses, namely, geometric or divergence loss, viscous drag loss, and chemical kinetics loss. Geometric loss results when a portion of the nozzle exit flow is directed away from the nozzle axis. Pressure difference with the change of altitude causes the viscous drag loss. The rapidly accelerating nozzle flow does not permit time for the gas to reach full chemical equilibrium.

A considerable number of numerical analysis has been undertaken to investigate these loss mechanisms and to overcome these losses. Method of Characteristics (MOC) is a computational procedure used to compute and simulate the two-dimensional nozzles. Both the conventional (conventional Bell) and unconventional (Aerospike and Expansion-deflection) nozzles can be the subject of consideration of the MOC.

Two-dimensional kinetics (TDK) computer program, which has been combined with a boundary layer module (BLM), is the industry standard JANNAF (Joint Army-Navy-NASA-Air Force) program which accounts for all three nozzle losses and is used to predict high nozzle efficiency for conventional bell nozzles.

Rao's method is another method used to obtain a shorter length nozzle and an improvement of over one percent in nozzle efficiency relative to a 15° cone. Apart from those, different research institutes and universities came up with modification of these numerical solutions and improved solutions for different objectives.

Theoretically, for ideal condition, a long nozzle is needed to maximize the geometric efficiency; whereas nozzle drag is reduced if the nozzle is shortened. If chemical kinetics is an issue, then the acceleration of exhaust gases at the nozzle throat should be slowed down by increasing the radius of curvature applied to the design of the throat region. In addition, a nozzle used for both supersonic wind tunnel and high speed vehicle should have a less space as well as should consume less weight.

Therefore, a minimum length supersonic nozzle is the subject of optimum nozzle design when other conditions like uniform flow at the exit with optimum

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Mach number is true for the design. Method of Characteristics is one of the most popular ways of designing such a supersonic nozzle.

2. THEORITICAL BACKGROUND

Assume that sonic flow exit at the throat and the area at the throat is A^*. The Mach number and the velocity at the throat denoted by M^* and u^*, respectively. At any other section of diverging section, the area the velocity and the Mach number are denoted as 'A', u, M, respectively.

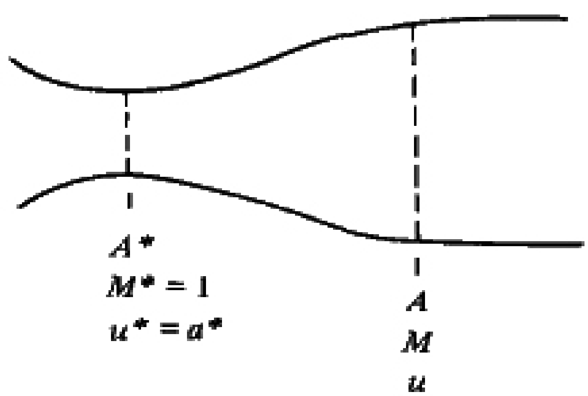

Figure-1. Geometry for the derivation of the area-mach number relation.

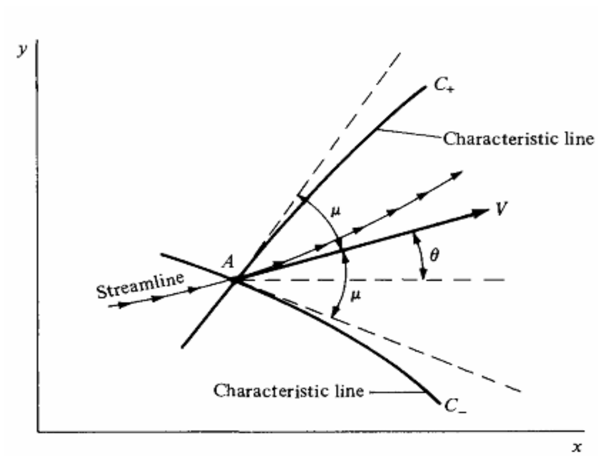

Figure-2. Left and right running characteristics lines through point A.

Consider a two-dimensional, steady, inviscid, supersonic, irrotational flow in xy space. The flow variables such as velocity, pressure, temperature, etc. are continuous throughout the xy space. There are points in which the derivatives of the flow field variables are indeterminate. The lines joining these points are called Characteristics line. Along this line the flow variables may even be discontinuous.

The exact governing equation for such a flow is:

$$\left[1-\left(\frac{1}{a^2}\right)\left(\frac{\partial\phi}{\partial x}\right)^2\right]\left(\frac{\partial^2\phi}{\partial x^2}\right)+\left[1-\left(\frac{1}{a^2}\right)\left(\frac{\partial\phi}{\partial y}\right)^2\right]$$

$$\left(\frac{\partial^2\phi}{\partial y^2}\right)-\left(\frac{2}{a^2}\right)\left(\frac{\partial\phi}{\partial x}\right)\left(\frac{\partial\phi}{\partial y}\right)\left(\frac{\partial^2\phi}{\partial x\partial y}\right)=0$$

Since $\frac{\partial\phi}{\partial x}=u$ and $\frac{\partial\phi}{\partial y}=v$

$$\left[1-\left(\frac{u^2}{a^2}\right)\right]\left(\frac{\partial^2\phi}{\partial x^2}\right)+\left[1-\left(\frac{v^2}{a^2}\right)\right]\left(\frac{\partial^2\phi}{\partial y^2}\right)-\left(\frac{2uv}{a^2}\right)\left(\frac{\partial^2\phi}{\partial x\partial y}\right)=0$$

The velocity potential and its derivatives are functions of x and y. Hence, for an exact differential,

$$df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$$

After relevant algebraic and trigonometric manipulation, the preceding equation reduces to,

$$\frac{dy}{dx}=\tan(\theta\mp\mu)$$

The above equation describes the direction of the two characteristic lines that run through point A. From the (Figure-2), it is apparent that the Characteristic lines are the Mach lines. The left and right running Characteristic lines are denoted as C_+ and C_-

The characteristics algebraic equations used are:

$\theta + v(M)$ = constant = k_- (along the C_- characteristic) 1(a)

$\theta - v(M)$ = constant = k_+ (along the C_- characteristic) 1(b)

3. METHODOLOGY

To expand a gas from rest to supersonic speed, a convergent- divergent nozzle should be used. Quasi-one dimensional analyses predict the flow properties as a function of x through a nozzle of specified shape. Although quasi-one dimensional analysis represents the properties at any cross section as an average of the flow over a given nozzle cross-section, it cannot predict both the three dimensional flow and the proper wall contour of the C-D nozzle. Therefore, quasi-two dimensional analysis is used to predict the proper contour for different conditions.

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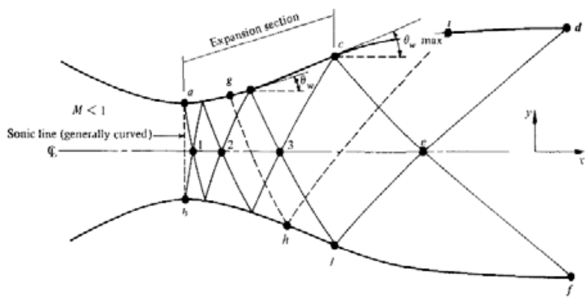

Figure-3. Schematic of supersonic nozzle design by the method of characteristics.

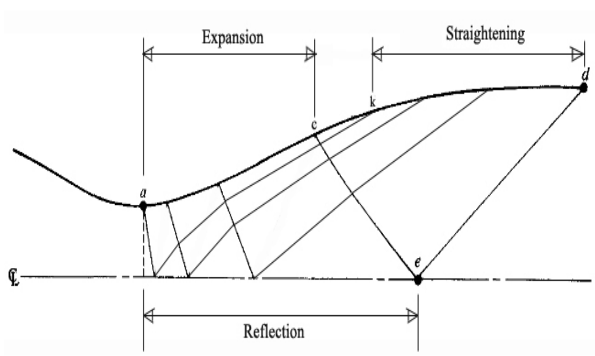

Figure-4. Schematic of supersonic nozzle design (above centerline) by the method of characteristics.

Due to the multidimensionality of the converging subsonic flow, the sonic line is gently curved. For simplicity, it is assumed that the sonic line is straight line. From the downstream of the dotted sonic line, the duct diverges.

Let θ_w represents the angle between the tangent at any point of the wall and x direction. In the section of the nozzle where θ_w increases is happened to call expansion section. Expansion section generates the expansion waves. These waves are propagated across the downstream and reflected from the opposite wall. The point C ends the expansion section and named as inflection point of the contour. In downstream of point C, the angle θ_w starts to decrease. Therefore, θ_w at point C is necessarily the maximum θ_w and denoted as $\theta_{w,\,max}$.

Downstream of $\theta_{w,\,max}$, θ_w decreases until the wall becomes parallel to the x-direction at point d. This section kd, named straightening section, is designed to cancel all the expansion waves generated by the expansion section. Flow beyond de is uniform and parallel at desired Mach number. Finally, due to the symmetry of the nozzle flow, the waves (characteristics) generated from the top wall act as if they are being reflected from the centerline. This symmetry allows one to consider only upper part of the nozzle.

For minimum length supersonic nozzle, the expansion section is shrunk to a point and, thereby, the expansion takes place through a centered Prandtl-Meyer wave emanating from a sharp-corner throat with an angle $\theta_{W\,max,\,ML}$, as sketched in Figure-3.

Let $\nu(M)$ be the Prandtl-Meyer function associated with the design exit Mach number. Hence, along the C_+ characteristic 'cb', $\nu = \nu(M) = \nu_c = \nu_b$. Now consider the C_- characteristic through points 'a' and 'c'.

At point 'c', from equation (1), $\theta_c + \nu_c = k_{-c}$

However, $\theta_c = 0$ and $\nu_c = \nu_M$, the above equation becomes,

$$\nu_M = k_{-c} \tag{2}$$

At point a, along the same C_- characteristic through ac, from equation (1),

$$\theta_{W\,max,\,ML} + \nu_a = k_{-a} \tag{3}$$

Since the expansion at point a is a Prandtl-Meyer expansion from initially sonic conditions, we know $\nu_a = \theta_{W\,max,\,ML}$; from equation (3),

$$\theta_{W\,max,\,ML} = \frac{1}{2} k_{-a} \tag{4}$$

However, along the same C_- characteristic $k_{-a} = k_{-c}$; hence equation (4) becomes,

$$\theta_{W\,max,\,ML} = \frac{1}{2} k_{-c} \tag{5}$$

Comparing equation (5) and (2),

$$\theta_{W\,max,\,ML} = \frac{1}{2} \nu_M$$

Which means for a minimum length nozzle the expansion angle of the wall downstream of the throat is equal to one-half the Prandtl-Meyer function for the design exit Mach number. Again, ν_M can be defined as,

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1} \tag{6}$$

3.1 Grid generation

Grid points used in calculation of Method of Characteristics are of two types: (1) internal points which are away from wall and (2) wall points.

3.2 Internal point

Consider the internal grid point 1, 2 and 3, as shown in (Figure-6). It is assumed that the location and flow properties at point 1 and 2 are known, whereas point 3 is the intersection of the C_- characteristic through point 1 and C_+ characteristic through point 2.

$$\theta_1 + \nu_1 = k_{-1} = k_{-3}$$

Similarly, at point 2, $k_{+2} = k_{+3} = \theta_2 - \nu_2$

From equation (4), for the point 3, it can be written,

$$\theta_3 + \nu_3 = k_{-3}$$
$$\theta_3 - \nu_3 = k_{+3}$$

From the above two algebraic equations, both θ_3 and ν_3 can be determined. The slope of the characteristic line joining points 1 and 3 is assumed to be $\frac{1}{2}(\theta_1 + \theta_3) - \frac{1}{2}(\mu_{1+}\mu_3)$. Similarly for characteristic line joining points 1 and 3, the slope is $\frac{1}{2}(\theta_2 + \theta_3) + \frac{1}{2}(\mu_{2+}\mu_3)$.

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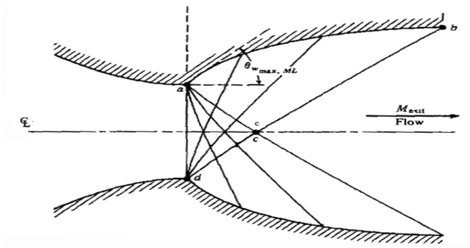

Figure-5. Schematic of minimum length supersonic nozzle design.

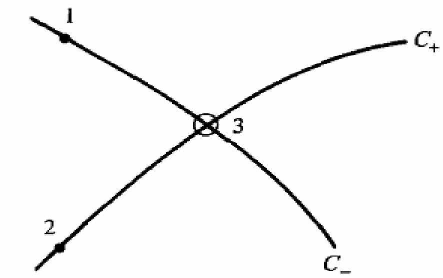

Figure-6. Characteristic mesh used for the location of point 3.

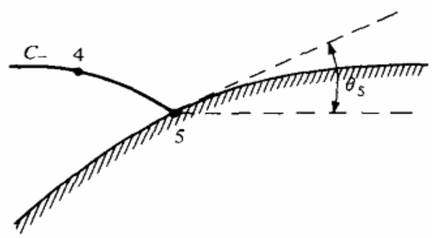

Figure-7. Wall point.

3.3 Wall point

Assume that we know the location and the properties at point 4. C_- Characteristic through point 4 intersects the wall at point 5. At point 5, the slope of the wall θ_5 is known. The wall properties at the wall point 5 can be obtained from the known properties.

$k_{-4} = k_{-5} = \theta_4 + \nu_4$

Again, $k_{-5} = \theta_5 + \nu_5$

The characteristic line is assumed to be straight-line between points 4 and 5 with an average slope $\frac{1}{2}(\theta_4 + \theta_5) - \frac{1}{2}(\mu_{4+} \mu_5)$.

4. RESULTS AND DISCUSSIONS

The design of minimum length supersonic nozzle is capable of producing minimum length nozzle by contracting the expansion section. With the contraction of the expansion section, the total length of the nozzle reduces. In the above design, the length of the supersonic nozzle is minimum, since the expansion section is minimum. In fact, the expansion section is contracted to a point at the end of the throat. The straightening section was right at the core of designing concentration. The straightening section is responsible for the uniform flow at outlet. Thereby, proper designing of the straightening section is imperative. In fact, straightening section is employed to control the way of interaction of the fluid coming out of the engine or the blower with the atmospheric fluid outside the nozzle. The efficiency and deficiency of the nozzle both depends on the design of the straightening section. All losses that occur in the actual flow conditions are by somewhat means relevant to the design of the straightening section. Any attempts to reduce these losses deals with analysis of the flow pattern in the straightening section.

Earlier, it was mentioned that the nozzle is being designed for the optimum exit Mach number. It should be recalled that the streamlines are turned away from the axis and, afterwards, they are turned back toward the axis in the diverging section. The turning away of the streamlines occurs at the expansion section, whereas the turning in of the streamlines occurs at the cancellation or the straightening section. The turning away angle is a function of local Mach number. Thereby, it is crystal clear to state that the last local as well as maximum turning away angle plays the vital role to define the exit Mach number.

The minimum length is achieved in the above output with the mesh generated by 10 characteristic lines. The reflection section begins from the length of 0.000847m and stretched to the length of 0.03352m. It should also be mentioned that the throat of the nozzle is 0.025m according to the design assumptions. The expansion waves are completely cancelled at the length of 0.0856m and height of 0.0256m. When the minimum length is achieved in the above output with the mesh generated by 30 characteristic lines then the reflection section begins from the length of 0.000847m and stretched to the length of 0.03326m. The expansion waves are completely cancelled at the length of 0.0873m and height of 0.0265m. In this output both the length and height have increased, but the reflection section has reduced. When the minimum length is achieved in the above output with the mesh generated by 50 characteristic lines then the reflection section begins from the length of 0.000847m and stretched to the length of 0.0331m. The expansion waves are completely cancelled at the length of 0.0875m and height of 0.0267m. In the above output both the length and height have increased slightly, but the reflection section has reduced.

Again when the minimum length is achieved in the above output with the mesh generated by 100 characteristic lines then the reflection section begins from the length of 0.000847m and stretched to the length of 0.03303m. The expansion waves are completely cancelled at the length of 0.0877m and height of 0.0269m. In the above output both the length and height have increased slightly, but the reflection section has further reduced. When the minimum length is achieved in the above output with the mesh generated by 200 characteristic lines then the reflection section begins from the length of 0.000847m and stretched to the length of 0.03301m. The expansion waves are completely cancelled at the length of 0.0878m and height of 0.0269m. In the above output both the length and height have increased slightly, but the reflection section has reduced. It is observed that with the refinement of mesh the result becoming steadier.

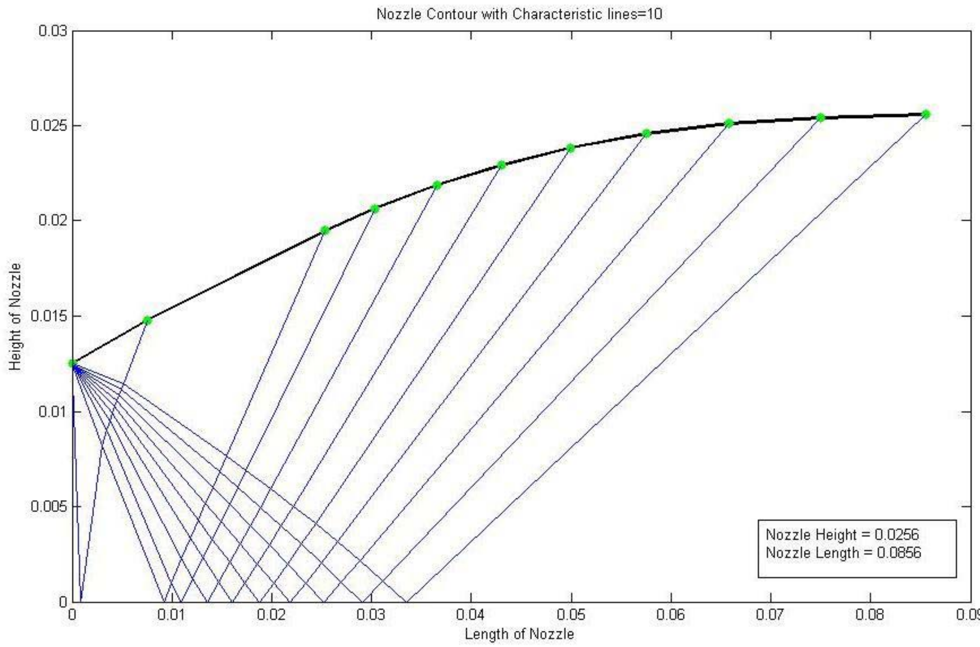

Figure-8. Numerical output for 10 characteristic line.

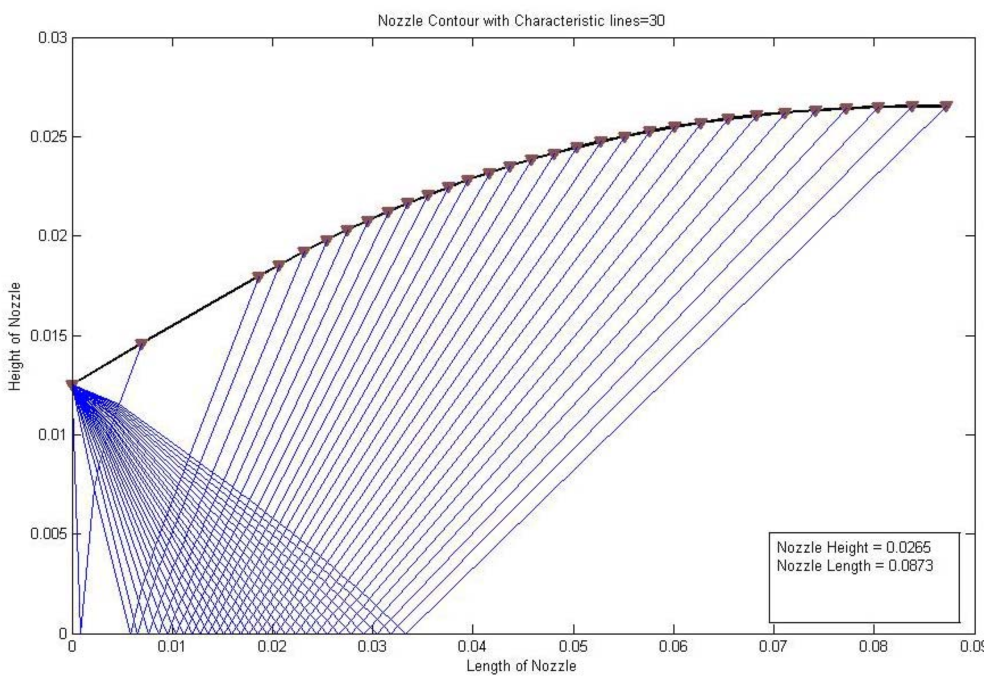

Figure-9. Numerical output for 30 characteristic line.

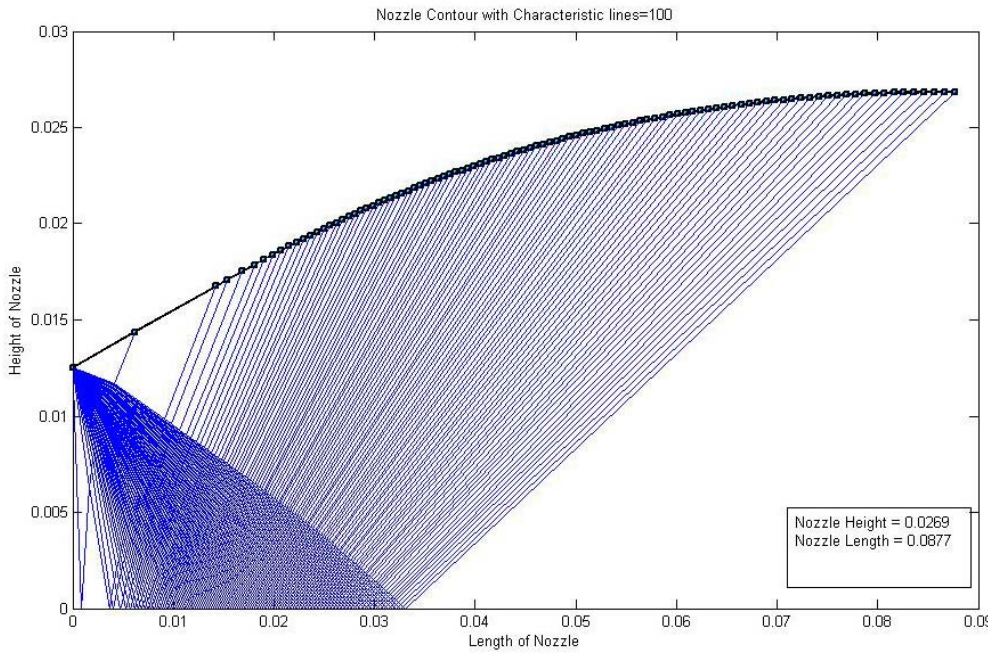

Figure-10. Numerical output for 50 characteristic lines.

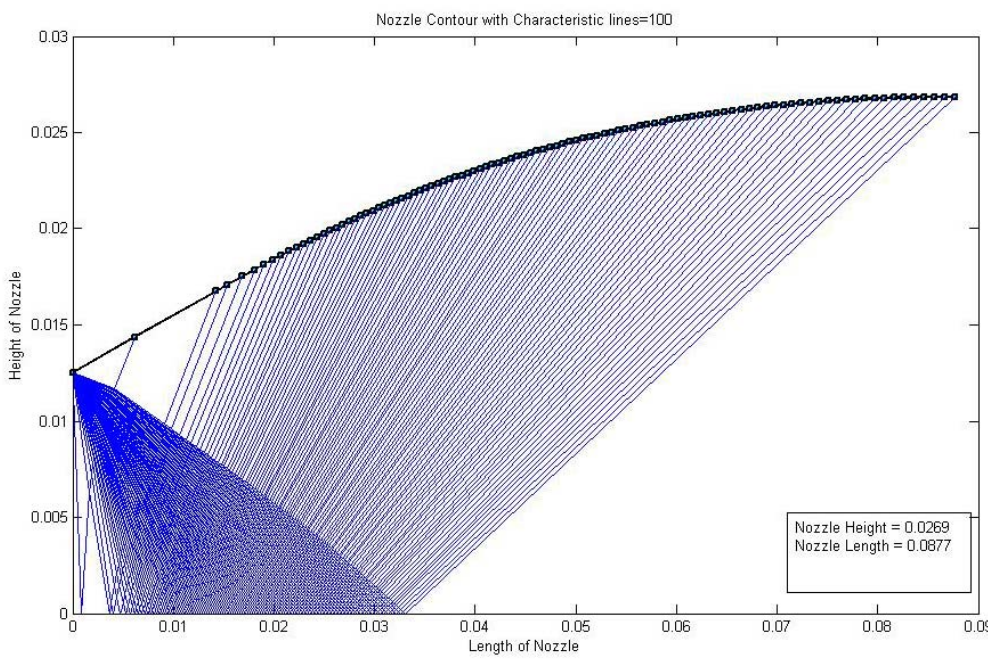

Figure-11. Numerical output for 100 characteristic lines.

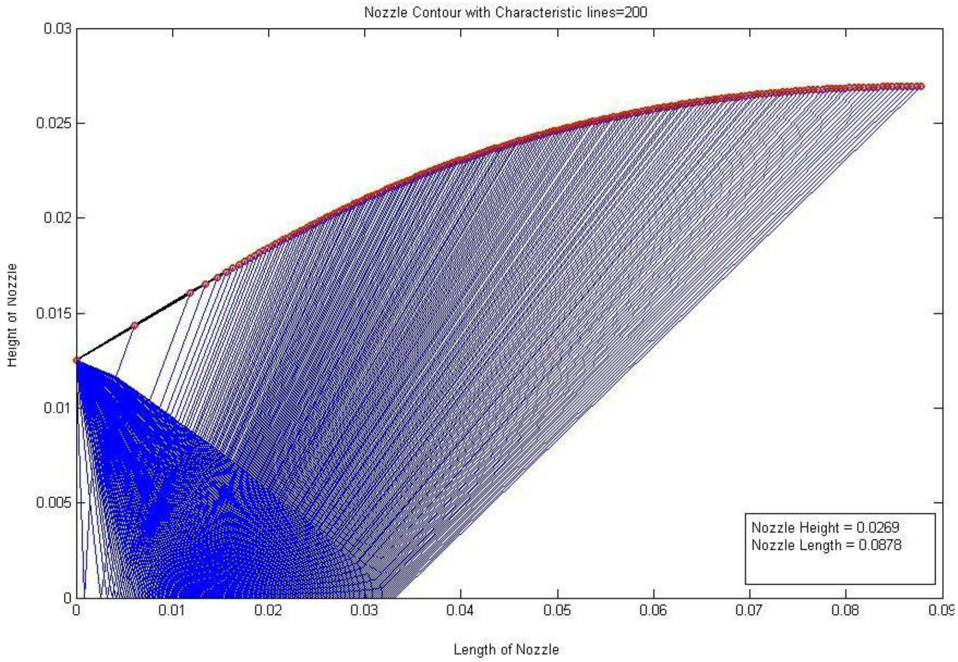

Figure-12. Numerical output for 200 characteristic lines.

With the help of equation (6), Mach number distribution can be defined for the exit flow. Mach number distribution is given by the following output:

- For the increase of the characteristic lines from 10 to 200, the nozzle length increases from 0.0856m to 0.0878m and the half of nozzle height increases from 0.0256m to 0.0269m. It has also been observed that the length and height of the nozzle is highly instable at low mesh. Both the length and height become fairly stable as the number of characteristic lines increases.
- The mesh has been made finer by increasing the number of characteristic lines 10 to 200 in five stages. Throughout the mesh increment process, it has become apparent that finer mesh produces rectified result. The evidence of the result rectification is set by nozzle contour smoothening.
- The last output describes the increment of the perfection of the straightening section design as the number of characteristic lines increases.

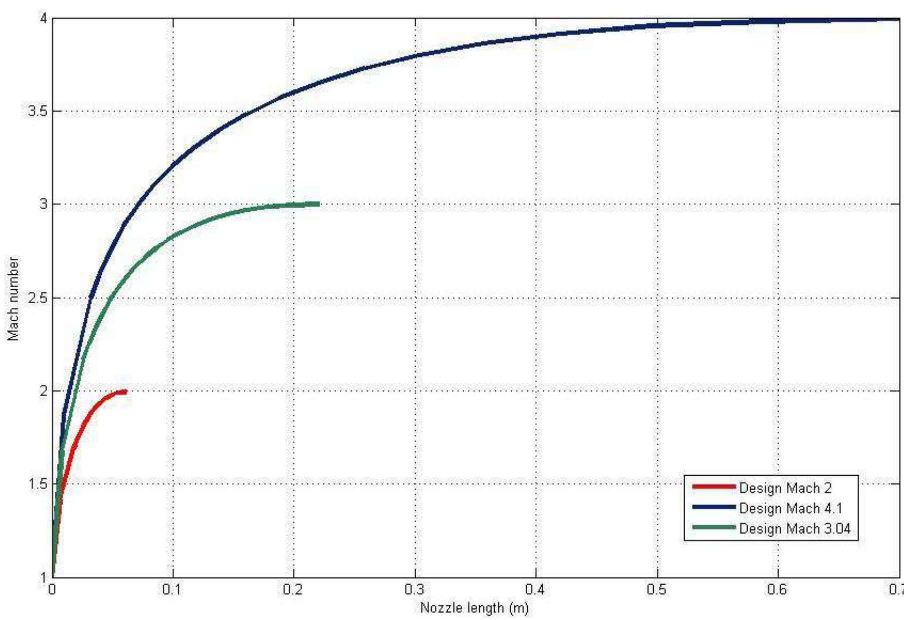

Figure-13. Mach number distribution along diverging section for design mach 2, 3.04, 4.1.

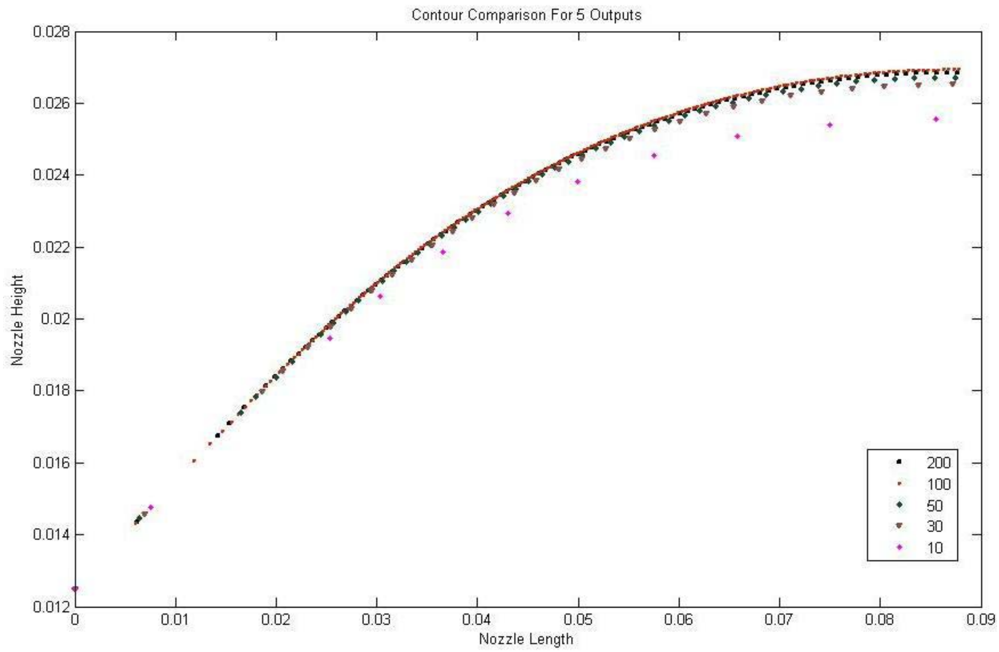

Figure-14. Numerical output for the design comparison with varying number of characteristic lines.

5. CONCLUSIONS

The complete design philosophy is developed and implemented for the ideal conditions. To address the actual conditions, the consideration should be made on account of the phenomena due to the viscous effect, pressure difference with respect to the back pressure, heat conduction and so on. The design presented here can be utilized to compare with the other nozzle designs regarding to the specific design conditions. The simulation program of the practical supersonic nozzle can be developed depending on this design with the considerations of the losses take place in real time.

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