FOSSEE Optimization Toolbox Workshop

Project FOSSEE Team

December 23, 2016

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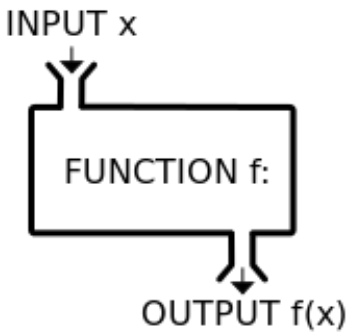
Optimization

- Select the best element from a given set of elements with respect to some criteria
- Function - relation between given input and output
- Every function input is related to exactly one output



- Mathematical function is used to define objective and constraints

Optimization Models

- Every mathematical model has two parts-
  - An Objective function
  - Zero, one or more constraint functions
- Example: While shopping with a fixed amount of cash and a list of items to buy
  - Objective - Buy as many items in the shopping list as possible (Maximize the value of purchase)
  - Constraint - Total expenditure should be less than or equal to the cash available

Optimization Models

We want to find minimum of a function f

$$\min_{x} \quad f(x)$$

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Constraints on x:

$$\begin{aligned} \text{subject to } r_1 \leq g_1(x) \leq s_1 \\ r_2 \leq g_2(x) \leq s_2 \\ \dots \\ r_m \leq g_m(x) \leq s_m \\ l_1 \leq x_1 \leq u_1 \\ l_2 \leq x_2 \leq u_2 \\ \dots \\ l_n \leq x_n \leq u_n \end{aligned}$$

Applications

- Industrial Engineering
  - Transportation Planning
  - Inventory Planning
  - Production Planning
- Engineering Design
  - Reactors and Processes
  - Devices
  - Plant layouts
- Finance
- Bioinformatics and Health Care
- Power Grids
- Agriculture
- ...

FOSSEE Optimization Toolbox

- Few optimization functions present in Scilab natively
- FOSSEE Optimization Toolbox (FOT) fills this gap
- Interfaces Scilab to COIN-OR libraries- SYMPHONY, IPOPT, CBC, BONMIN
- Available on Scilab 5.5.0 and later versions
- Available from Scilab atoms website
- Function syntax and function call matches Matlab Optimization Toolbox function equivalent
- More details- http://scilab.in/fossee-scilab-toolbox/optimization-toolbox

COIN-OR

- Computational Infrastructure for Operations Research
- Open-source solvers and libraries for a variety of optimization problems
- http://www.coin-or.org
- Leading researchers from universities worldwide

Different solvers for different problems

- When all functions are linear – Linear Programming (LP) – Solver: CLP
- When the objective function is quadratic and constraint functions are linear – Quadratic Programming (QP) – Solver: QP-IPOPT
- When all functions are linear, but some variables must be integers – Integer Linear Programming (ILP) – Solver: Symphony
- When some functions are nonlinear – Nonlinear Programming (NLP) – Solver: Ipopt
- Other solvers: BONMIN for MINLP, CBC for MILP, semi-infinite programming, ...

Linear Programming

- Optimization of a linear objective function, subject to linear equality and linear inequality constraints. e.g.

$$\begin{aligned} \text{Min} \quad & c_1 x_1 + c_2 x_2 \\ \text{s.t.} \quad & a_{11} x_1 + a_{12} x_2 \leq b_1 \\ & a_{21} x_1 + a_{22} x_2 \leq b_2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- General form:

$$\begin{aligned} \text{Min} \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \end{aligned}$$

```
[xopt, fopt, exitflag] = linprog(c, A, b)
```

Linear Programming – More General Forms

$$\begin{aligned} \text{Min} \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & Hx = g \end{aligned}$$

```
[xopt, fopt, exitflag] = linprog(c, A, b, H, g)
```

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$$\begin{aligned} \text{Min} \quad & c^T x \\ \text{s.t.} \quad & Ax \leq b \\ & Hx = g \\ & l \leq x \leq u \end{aligned}$$

```
[x, fopt, exitflag] = linprog(c, A, b, H, g, l, u)
```

Linear Programming – More General Forms

If your LP is not of the above form

- A $\geq$ constraint can be converted to $\leq$ constraint by multiplying it with (-1)
- If the objective is to maximize, then multiply it with (-1) and then minimize ...
- then the optimal solution would be the same, but the objective value of max is the negative of the optimal value min
- Remember `linprog` only minimizes

Getting Help

- FOT comes with inbuilt help and demos
- Press 'F1' for general help
- Or get help on individual functions
  `help linprog`
- For demos and examples: '?' → Scilab Demonstrations → FOSSEE_Opimization_Toolbox
- Scilab forum at `http://forums.fossee.in`
- We can answer your questions in the forum

Quadratic Programming

- Objective is quadratic and all constraints are linear, e.g.

$$\begin{aligned} \text{Min } 2x_1^2 + 4x_2^2 - 4x_1x_2 + x_3^2 + 2x_1x_3 + x_4^2 \\ \text{s.t. } x_1 + x_2 + x_3 + x_4 = 1, \\ 3x_1 - 2x_2 + x_4 \leq 4, \\ x_2 \geq 0. \end{aligned}$$

- Convex when H is positive semidefinite (all eigen values are ≥ 0)
- `qpipopt` finds a local solution – using IPOPT
- Starting point can be provided

Quadratic Programming – General Form

- In general:

$$\text{Min } \frac{1}{2}x^T Hx + c^T x$$

$$\text{s.t. } p \leq Ax \leq q$$

$$l \leq x \leq u$$

```
[xopt, fopt, exitflag] = qpipopt(n, m, H, c, l, u,
A, p, q)
```

Nonlinear Programming

- The objective function or constraints could be nonlinear, e.g.

$$\begin{aligned} \text{Min } & 2x_1^2 + 4x_2^4 - 4x_1x_2 \\ \text{s.t. } & x_1^2 + x_2 \leq 100, \\ & x_1 - \log(x_2) \leq 4, \\ & 3x_1 - 2x_2 \leq 10, \\ & x_2 \geq 1. \end{aligned}$$

- Use `fmincon` function – local solver based on IPOPT
- Nonlinear functions can not be denoted using matrices
- User needs to write their own Scilab routines to evaluate nonlinear functions
- Pass a reference to these subroutines to `fmincon`

Nonlinear Programming

- Write one Scilab function `objfun` that evaluates the objective function at a given point

```
function y=objfun(x)
y = 2*x(1)*x(1) + 4*x(2)^4 - 4x(1)x(2)
endfunction
```

- Write one Scilab function `confun` that evaluates all the constraint functions

```
function [c,ceq]=confun(x)
c = [x(1)*x(1) + x(2),
     x(1) - log(x(2))]
ceq = []
endfunction
```

- `[x, fopt, exitflag] = fmincon(objfun, x0, A, b, Aeq, beq, confun)`

Thank you