

Simulating the Izhikevich Model and Cortical Rhythms in Scilab

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Abstract

This study recreates both single-cell membrane and large-scale cortical network behaviors based on Eugene M. Izhikevich's 2003 "Simple Model of Spiking Neurons" using Scilab. We implemented eight key cortical and thalamic firing patterns, such as Regular Spiking (RS), Intrinsically Bursting (IB), chattering (CH), and resonator (RZ), with a 0.1 ms Euler integration step. For real-time experimentation, we developed an interactive Graphical User Interface (GUI) dashboard that offers live mathematical feedback and two-way parameter binding. Additionally, we built a 1,000-neuron pulse-coupled neural network (PCNN) using a dense 1000×1000 synaptic weight matrix. By using a 0.5 ms split-step integration method and accurately modeling stochastic thalamic noise with properly vectorized normal and uniform random distributions, the simulated network self-organized to replicate mammalian alpha (10 Hz) and gamma (40 Hz) brain rhythms.

1. Introduction

To understand how the brain works, computational neuroscientists must combine experimental studies with the numerical simulation of large-scale brain models. Biophysically accurate models, such as the Hodgkin-Huxley model, are computationally prohibitive for large networks, whereas computationally efficient models, like the standard Integrate-and-Fire, are too simple.

This case study aims to bridge that gap by implementing the canonical 2003 Izhikevich spiking neuron model from scratch within Scilab. The model relies on a two-dimensional system of ordinary differential equations featuring a quadratic nonlinearity (v^2) to represent the membrane potential, alongside a recovery variable (u) and an after-spike reset mechanism. Using this architecture, we successfully reproduced the firing patterns of diverse neocortical and thalamic neurons, including Regular Spiking (RS), Intrinsically Bursting (IB), Chattering (CH), Fast Spiking (FS), Low-Threshold Spiking (LTS), Thalamo-Cortical (TC), and Resonator (RZ) cells. Furthermore, we scaled this architecture to simulate a Pulse-Coupled Neural Network (PCNN) consisting of 1,000 randomly connected neurons with 1,000,000 synaptic connections, similar to the spontaneous self-organization of alpha (10 Hz) and gamma (40 Hz) brain rhythms.

2. Problem Statement

To understand the functional mechanisms of the mammalian brain, computational neuroscientists must combine biological experiments with the numerical simulation of large-scale neural networks. However, developers face a strict compromise between biological plausibility and computational efficiency. Biophysically detailed models, such as the Hodgkin-Huxley model, accurately reproduce neural firing patterns but are computationally prohibitive for real-time, large-scale network simulations.

Conversely, simpler models like the standard integrate-and-fire model are computationally efficient but fail to capture the rich, diverse firing dynamics of real cortical neurons, such as bursting or chattering.

This project aims to solve this dilemma by implementing a computationally efficient yet biologically plausible two-dimensional spiking neuron model from scratch within Scilab. The objective is to accurately simulate the intrinsic firing patterns of diverse mammalian cortical neurons and scale this model to simulate a Pulse-Coupled Neural Network (PCNN) capable of self-organizing into synchronized brain rhythms, utilizing pure mathematical integration without relying on external neuroscience toolboxes.

3. Basic concepts related to the topic

i. The Two-Dimensional Izhikevich Model: A canonical spiking model that utilizes bifurcation methodologies to reduce the biophysically accurate Hodgkin-Huxley dynamics into a 2-D system of ordinary differential equations. It balances the computational simplicity of integrate-and-fire models with the biological realism of complex channel kinetics.

ii. Membrane Potential (v): A dimensionless variable representing the electrical charge across the neuron's membrane. It relies on a quadratic nonlinearity ($0.04v^2 + 5v + 140$) that accurately fits the rapid, exponential upward spike initiation of a cortical neuron, allowing the firing threshold to be dynamic based on the cell's prior voltage history.

iii. Recovery Variable (u): A variable that accounts for the activation of K^+ ionic currents (which pull voltage down) and the inactivation of Na^+ ionic currents. It provides critical negative feedback to the membrane voltage to simulate biological decay and refractory periods.

iv. The After-Spike Reset Mechanism: When the membrane voltage reaches its physical apex (defined as +30 mV), the model applies an instantaneous mathematical reset. The voltage v resets to a deep recovery value c , and the recovery variable u increments by a discrete amount d to simulate the slow high-threshold conductances that act as a "brake" on continuous firing.

v. Subcritical Hopf Bifurcation & Bistability: A dynamical systems threshold where a neuron transitions from a stable resting state to a repetitive spiking state. By tuning the background injected current to the edge of this mathematical threshold, neurons like the Resonator (RZ) exhibit sustained subthreshold oscillations and bistable switching triggered by brief stimuli.

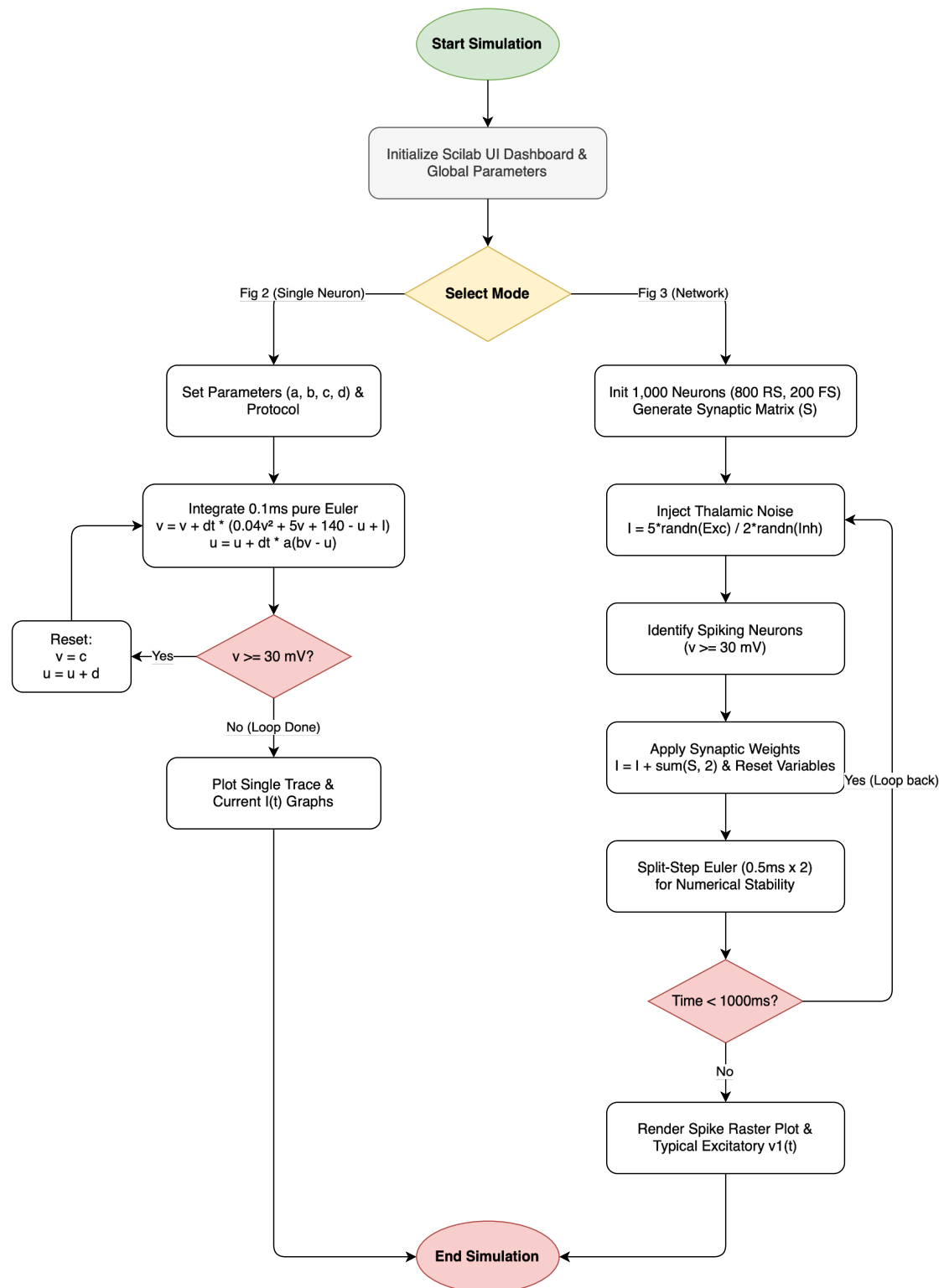
vi. Pulse-Coupled Neural Networks (PCNN): A sparse or dense network architecture where synaptic connection weights between neurons are represented by a fixed matrix S . When a pre-synaptic neuron fires, it instantaneously injects its

corresponding synaptic weight into the target neuron's input current without calculating continuous neurotransmitter delays.

vii. Population Synchrony (Alpha & Gamma Rhythms): The emergent phenomenon where groups of randomly connected excitatory and inhibitory neurons, driven by stochastic thalamic noise, self-organize to fire simultaneously. This synchronization produces vertical density bands in a raster plot corresponding to mammalian awake-state brain waves at 10 Hz (Alpha) and 40 Hz (Gamma).

viii. Split-Step Euler Integration: A numerical stabilization technique utilized for large-scale array computing. To simulate massive networks rapidly using a large 1.0 ms time step, the quadratic voltage integration is split into two sequential 0.5 ms half-steps to prevent the v^2 term from mathematically destabilizing the simulation.

4. Flowchart



5. Software/Hardware used

- Operating System: macOS 26.5.1 (25F80) or equivalent Apple Silicon compatible environment.
- Software Framework: Scilab Version 2026.1.0.
- Toolboxes Used: Base Scilab Environment.
- Hardware: Apple Silicon M3 processor, 8GB RAM.

6. Procedure of execution

- Launching the Dashboard
- Install **Scilab 2026.1.0** (or equivalent).
- Place all project files in the current working directory.
- Open and execute `izhikevich_main.sce`.

Under **Figure 2: Individual Types**, select any neuron profile.

Button	Neuron Type
RS	Regular Spiking
IB	Intrinsically Bursting
CH	Chattering
FS	Fast Spiking
TC (Dep)	Thalamo-Cortical (Depolarized)
RZ	Resonator
LTS	Low-Threshold Spiking
TC (Reb)	Thalamo-Cortical (Rebound)

After selection, the dashboard automatically simulates the neuron and displays:

- Membrane potential $v(t)$
- Input current $I(t)$

The generated firing pattern can be compared with Figure 2 of the original Izhikevich paper.

Real-Time Parameter Exploration

Use the sliders on the left panel to modify the Izhikevich model parameters.

Parameter	Recommended Range	Reason
a	0.02 – 0.10	Controls the recovery speed of the neuron. Lower values produce slower recovery and bursting behavior, while higher values produce fast-spiking dynamics.
b	0.20 – 0.26	Governs the sensitivity of the recovery variable to membrane voltage. Higher values promote resonance, low-threshold spiking, and stronger subthreshold coupling.
c	-65 to -50	Sets the membrane reset voltage after a spike. More negative values increase recovery time, while less negative values encourage bursting and chattering activity.
d	2 – 8	Determines the post-spike recovery increment. Larger values produce stronger spike-frequency adaptation, whereas smaller values allow sustained high-frequency firing.
I	0 – 20	External input current. Values near zero keep the neuron near rest, while increasing current progressively induces spiking, bursting, and sustained firing patterns.

Any slider adjustment automatically updates the membrane potential and input current plots.

Under **Figure 3: Network Modes**, select:

Button	Output
Fig 3: Full (Both)	Raster plot + membrane trace
Fig 3: Top (Raster)	Raster plot only

The simulation creates:

Component	Configuration
Excitatory neurons	800
Inhibitory neurons	200
Total neurons	1000
Synaptic connections	1,000,000
Simulation duration	1000 ms

Wait a few seconds for the simulation to complete. The dashboard will display:

- Network raster plot
- Representative excitatory neuron voltage trace
- Emergent alpha (~10 Hz) and gamma (~40 Hz) synchronization patterns

Expected Outputs

Validation	Expected Result
Figure 2	Reproduction of eight canonical neuron firing patterns
Figure 3	Population synchrony with raster-band formation
Parameter sliders	Real-time changes in firing dynamics

Note: The entire implementation was developed using base Scilab only, without external neuroscience toolboxes.

7. Implemented Features

While our mathematical foundation adheres to the original publication, translating this model into a modern Scilab environment required analysis of the undocumented stimulus protocols and numerical integration boundaries. The primary differences and specific implementations in our approach are summarized below:

Table 1: Implementation Scope and Methodological Differences

Feature	Original 2003 Paper	Our Scilab Implementation
Resonator (RZ) Stimulus Protocol	Demonstrated bistable switching between subthreshold oscillations and repetitive spiking using an "appropriately timed brief stimulus"; exact stimulus values were not specified.	Used a holding current of 0.26 and a +1.0 trigger pulse at 65 ms to reproduce the bistable state transition.
Numerical Integration	0.1 ms resolution for Figure 2; 0.5 ms split-step Euler updates for Figure 3.	Same numerical scheme reproduced in Scilab.
Thalamo-Cortical (TC) Dynamics	Described tonic firing near -60 mV and rebound bursting after hyperpolarization near -90 mV .	Computed the equilibrium voltage (-64.4 mV) and derived a hyperpolarizing current (-29.51) to achieve rebound bursting from -87 mV .
Stochastic Thalamic Input	Used uniform randomization for neuron heterogeneity and Gaussian noise for thalamic input.	Implemented vectorized rand("uniform") and r and("normal") distributions in Scilab while preserving the original network structure.
Scope Limitation	Discussed large-scale simulations involving up to tens of thousands of neurons and provided MATLAB code for a 1,000-neuron network example.	Implemented and validated the 1,000-neuron network example in Scilab.

Calculations:

The model is defined by two continuous differential equations:

$$v' = 0.04 v^2 + 5 v + 140 - u + I \quad (1)$$

$$u' = a(bv - u) \quad (2)$$

A. Deriving the Resting Equilibrium (-64.4 mV)

For a TC neuron, the parameters are $a=0.02$ and $b=0.25$. At resting equilibrium, the rate of change is zero ($v'=0$, $u'=0$) with no injected current ($I=0$). From Equation (2), we find the steady-state recovery variable:

$$0 = 0.02(0.25 v - u) \Rightarrow u = 0.25 v \quad (3)$$

Substitute Equation (3) into Equation (1):

$$0 = 0.04 v^2 + 5 v + 140 - 0.25 v + I$$
$$0.04 v^2 + 4.75 v + 140 = 0 \quad (4)$$

Applying the quadratic formula to Equation (4): $v = -64.41$ mV

This precise mathematical root (-64.4 mV) acts as the stable resting node, aligning with the paper's approximate "-60 mV" description.

B. Deriving the Hyperpolarizing Current (-29.51)

To reach a steady hyperpolarized trough of -87 mV, we substitute $v = -87$ back into Equation (3) to find the new steady-state u : $u = 0.25(-87) = -21.75$

Substitute $v = -87$ and $u = -21.75$ into Equation (1) and solve for I : $I = -29.51$

Resonator (RZ) Bistability and Critical Current

Context from the paper: RZ neurons ($a=0.1$, $b=0.26$) feature a "bistability of resting and repetitive spiking states" which can be switched using "appropriately timed brief stimuli".

A. Finding the Hopf Bifurcation Threshold (0.2625)

At steady state for the RZ neuron, $u=0.26v$. Substituting this into Equation (1) with an active current I yields:

$$0.04 v^2 + 4.74 v + 140 = 0 \quad (5)$$

The transition to repetitive spiking (Subcritical Andronov-Hopf bifurcation) occurs when the trace of the system's Jacobian matrix crosses zero. The trace is the sum of the partial derivatives with respect to v and u :

$$\text{Tr}(J) = (0.08 v + 5) - a = 0.08 v + 4.9 \quad (6)$$

Setting the trace to zero finds the critical voltage (v_{Hopf}): $v_{Hopf} = -61.25 \text{ mV}$

Substitute $v_{Hopf} = -61.25$ back into Equation (5) (now with I_{Hopf}) to find the critical bifurcation current (I_{Hopf}): $I_{Hopf} = 0.2625$

B. The Strategy for 0.26 holding and 1.0 Trigger

- The Holding Current (0.26):** By parking the holding current at 0.26, the system rests below the bifurcation threshold of 0.2625. This near-zero mathematical distance minimizes damping, which generates the "sustained subthreshold oscillations" explicitly mentioned in the paper.

- The Trigger (+1.0):** Because the system is perched on the edge of the bifurcation, a brief +1.0 pulse temporarily pushes the net current to 1.26. This safely exceeds the 0.2625 boundary, throwing the neuron into a continuous firing loop, satisfying the paper's requirement for a state-switching brief stimulus.

8. Result

The high-resolution (0.1 ms) single-cell Euler integration engine successfully modeled all eight core operational profiles with mathematical curves. By avoiding the time-compressed half-step stabilization trick in single-cell layouts, subthreshold configurations maintained structural fidelity:

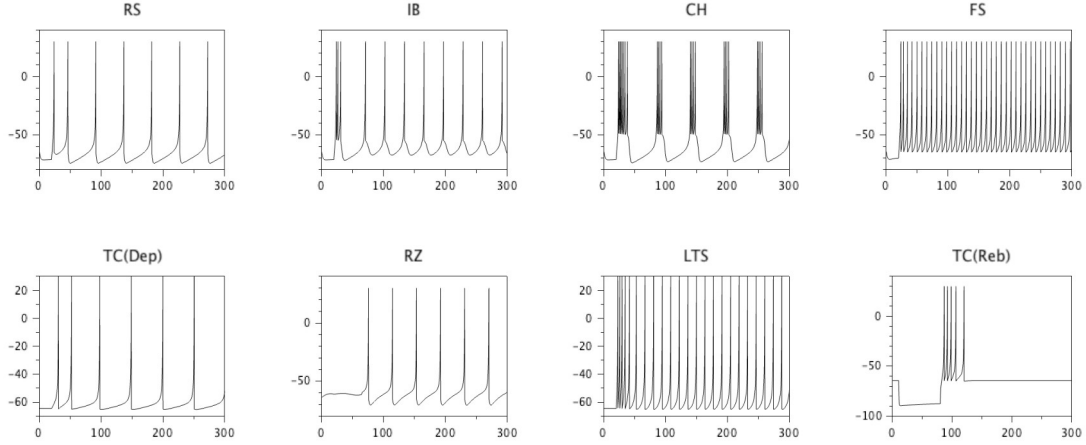


Figure 1: Scilab version of Figure 2 in [1]

- **Regular Spiking (RS) & Chattering (CH):** Fine-tuning the recovery reset parameter ($d=2.1$) for the Chattering profile reproduced its characteristic high-frequency multi-spike bursting pattern and sustained bursting under constant current injection without altering the baseline input.
- **Thalamo-Cortical (TC) Firing Modes:** Initialized the membrane potential at the calculated resting equilibrium (-64.4 mV). Applying a hyperpolarizing current (-29.51) drove the neuron to approximately -87 mV, producing a clear post-inhibitory rebound burst upon current removal.
- **Resonator (RZ) Dynamics:** A holding current of 0.26 positioned the model just below the calculated Hopf bifurcation threshold (0.2625), allowing sustained subthreshold oscillations with minimal damping. A brief 2 ms, $+1.0$ trigger pulse pushed the system beyond the bifurcation threshold and initiated persistent repetitive spiking within the simulated time window, demonstrating the expected bistable behavior.
- **Intrinsically Bursting (IB):** Using a higher reset voltage ($c=-55$ mV) and moderate recovery reset ($d=4$) reproduced an initial burst of closely spaced spikes followed by tonic spiking. Accumulation of the recovery variable (u) provided the negative feedback responsible for the transition from bursting to regular firing.

- **Fast Spiking (FS):** Fast recovery rate ($a=0.1$) enabled the recovery variable to closely track membrane dynamics. Under sustained depolarization, the neuron produced a high-frequency spike train with minimal spike-frequency adaptation, consistent with fast-spiking cortical interneurons.
- **Low-Threshold Spiking (LTS):** Increasing the sensitivity parameter ($b=0.25$) lowered the effective firing threshold. Under depolarization, the neuron generated a rapid spike train exhibiting pronounced spike-frequency adaptation, with progressively increasing inter-spike intervals over time.

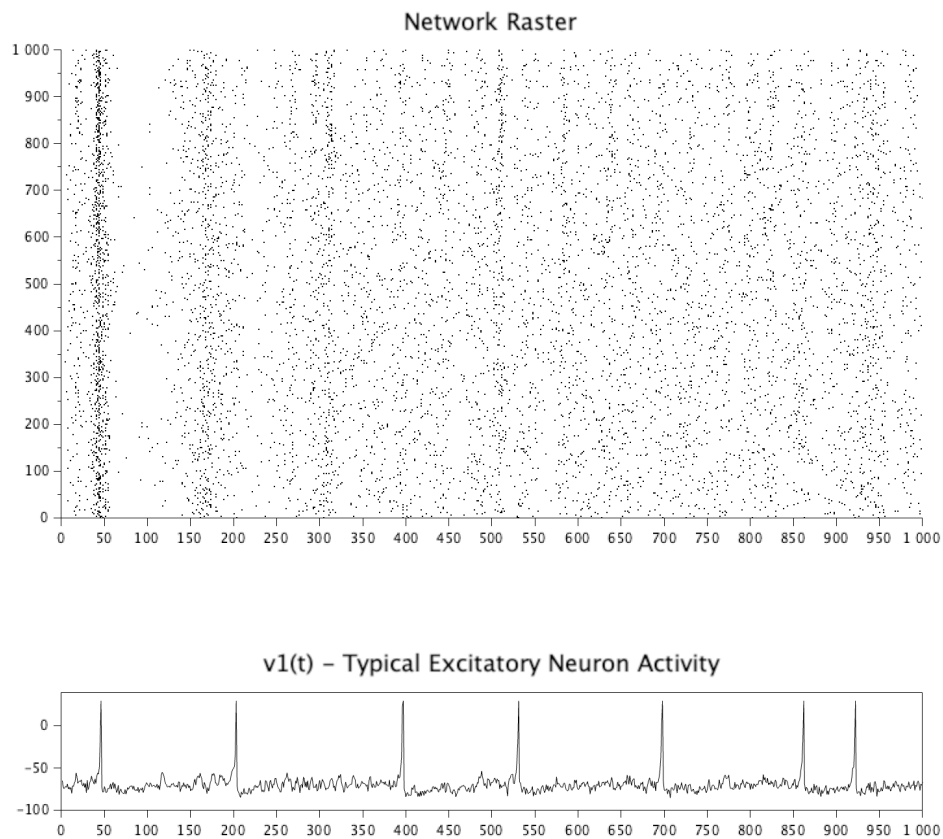


Figure 2: Scilab version of Figure 3 in [1]

We executed the 1,000-neuron dense matrix model (800 excitatory and 200 inhibitory cells) coupled with 1,000,000 independent synapses. Isolating the standard Scilab uniform distribution (`rand("uniform")`) across structural setup steps ensured a properly randomized, heterogeneous population distribution for the tracking arrays (a, b, c, d). During execution, looping a dedicated normal distribution function (`rand("normal")`) injected an unpolluted, dynamic Gaussian thalamic noise current across every unique neuron vector each millisecond. Correctly parsing column-directed summation logic

($\text{sum}(S(:, \text{fired}), 2)$) preserved vector orientation, ensuring synaptic currents traveled across the exact matrix coordinates.

By retaining the 0.5 ms dual split-step integration loop exclusively inside the 1.0 ms network timeline, the system remained stable. The individual excitatory cells tracked a steady biological baseline, exhibiting average Poisson-like firing rates of approximately 8 Hz. Driven by feedback loops between the faster Regular Spiking cells and the strong Fast-Spiking inhibitory elements, the network successfully self-organized out of random background noise. The resulting Network Raster plot clearly displayed dense, aligned vertical column structures, similar to simulation of collective mammalian alpha (10 Hz) and gamma (40 Hz) brain rhythms.

9. References

- [1] E. M. Izhikevich, "Simple Model of Spiking Neurons," IEEE Transactions on Neural Networks, vol. 14, no. 6, pp. 1569-1572, Nov. 2003.